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Finite Element Analysis of a cracked beam subjected to a moving load with Piezoelectric patches

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Abstract: *This article presents a finite element formulation for the application of piezoelectric patches for the repair of a cracked Timoshenko beam subjected to a moving load. Conceptually, an external voltage is applied to actuate a piezoelectric patch bonded on the beam to effect closure of the crack so that the singularity induced by the crack tip may be eliminated. This has the effect of altering the electromechanical admittance signature, extracted at the electrical terminals of specific piezoelectric patch, considered as an admittance calculating sensor patch, towards that of the healthy beam, which is the criterion concept used for the repair in this paper. The beam is discretized in to a number of simple elements with four degrees of freedom each. The inertial effects of moving load and piezoelectric patch are incorporated in to a finite element model. Some numerical results are shown to present the crack effects.*

Keywords: *Finite element, Cracked beam, Moving load, Piezoelectric, Timoshenko beam.*

1. Introduction

The stiffness of structural component will be reduced when they are subjected to cracks and consequently their natural frequencies too. The reality of affecting a crack or local defect on structural member dynamic response is recognized since years ago[1-5]. Defects in a structure can cause changes in mass distribution and damping properties. Many works in this field are related to cracked beams subjected to different boundary conditions. Dimarogonas [6] presented a review of the dynamics of cracked structures. Chondros et al [7] developed a complete cracked-beam vibration theory for the transverse vibration of a cracked Euler–Bernoulli beam with single-edge or double-edge open cracks. The influence of moving load on structures is a problem in the engineering field, a moving load will produce larger deformations and higher stresses ratio in this case when an equivalent load is applied statically. Many of studies had also been done on this field. That can be noted to Chen [8] who showed how a finite element could be used to efficiently model bridge under moving loads. Also Todd and Vohra [9] presented a theoretical method to reconstruct the beam shape under static or moving load. However, not so many studies were reported on the effect of cracks for moving load problems. Mahmoud [10] used an equivalent static load approach to determine the stress intensity factors for a crack in a beam subjected to a moving load. Parhi and Behera [11] used the Runge–Kutta method to determine the deflection of a cracked circular shaft subjected to a moving mass. Mahmoud and Zaid [12] used an iterative modal analysis approach to find the response of a cracked simply supported beam subjected to a moving mass. Bilello and Bergman [13] developed a theoretical and experimental study on the response of a damaged Euler-Bernouli beam traversed by a moving mass. The investigation in this study presents a numerical method for a cracked simply supported beam subjected to a concentrated moving load. The method used in Diamarogonas [6], obtained stiffness matrix for crack element. The repair of cracked components in structures using a composite patch to increasing their stiffness has been studied in many practical and academic cases. Attractive properties of high stiffness, strength, and lightweight are considered for composite patch [14]. The piezoelectric materials which have been studied for years can be applied as an alternative. It has the benefits of effecting the repair actively. This paper is organized in the following way. In section 2, finite element formulation for the application of piezoelectric patches for the repair of a cracked Timoshenko beam subjected to a moving load is presented. In section 3 numerical results are presented for case study and conclusions are in section 4.

2. Mathematical Modeling of Beam

A Timoshenko simply supported beam of length l_b with crack located and a concentrated moving load P with constant speed V is shown in Fig.1. The dimensions of the uniform cross section of the beam are: width b, height h,crack depth a, l_c is distance between the right hand side end node n and the crack location and ζ is distance of the crack from left hand side end node 1.

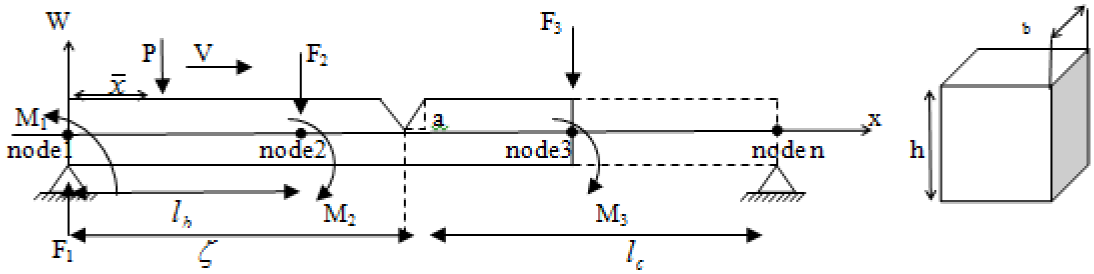


Fig. 1: A simply supported beam element.

2.1. Finite Element modeling of the healthy beam element

A simply supported Timoshenko beam subjected to a moving load with velocity V along the beam is shown in Fig.1. The Timoshenko beam is discretized into a number of simple elements with equal length each. The beam element consists of two nodes and each node has two degrees of freedom, i.e. vertical displacement w^e and bending rotation θ^e as follows:

$$w^e(x,t) = [N_{w1} N_{w2} N_{w3} N_{w4}]\{q\} = [N_w]\{q\} \tag{1}$$

$$\theta^e(x,t) = [N_{\theta 1} N_{\theta 2} N_{\theta 3} N_{\theta 4}]\{q\} = [N_\theta]\{q\} \tag{2}$$

Where $[N_w]$,$[N_\theta]$are the shape functions and $\{q\}$ is the vector of displacements and slopes are written as [15]

$$N_{w1} = \frac{2\left(\frac{\bar{x}}{l_b}\right)^3 - 3\left(\frac{\bar{x}}{l_b}\right)^2 - \varphi\left(\frac{\bar{x}}{l_b}\right) + (1+\varphi)}{(1+\varphi)} \tag{3}$$

$$N_{w2} = \frac{l_b\left\{\left(\frac{\bar{x}}{l_b}\right)^3 - \left(2+\frac{\varphi}{2}\right)\left(\frac{\bar{x}}{l_b}\right)^2 + \left(1+\frac{\varphi}{2}\right)\left(\frac{\bar{x}}{l_b}\right)\right\}}{(1+\varphi)} \tag{4}$$

$$N_{w3} = \frac{-2\left(\frac{\bar{x}}{l_b}\right)^3 + 3\left(\frac{\bar{x}}{l_b}\right)^2 + \varphi\left(\frac{\bar{x}}{l_b}\right)}{(1+\varphi)} \tag{5}$$

$$N_{w4} = \frac{l_b\left\{\left(\frac{\bar{x}}{l_b}\right)^3 - \left(1-\frac{\varphi}{2}\right)\left(\frac{\bar{x}}{l_b}\right)^2 - \left(\frac{\varphi}{2}\right)\left(\frac{\bar{x}}{l_b}\right)\right\}}{(1+\varphi)} \tag{6}$$

$$N_{\theta 1} = \frac{6\left\{\left(\frac{\bar{x}}{l_b}\right)^2 - \left(\frac{\bar{x}}{l_b}\right) - \frac{\varphi(1+\varphi)}{6} + \frac{(1+\varphi)^2 l_b}{6}\right\}}{(1+\varphi)l_b} \tag{7}$$

$$N_{\theta 2} = \frac{3\left(\frac{\bar{x}}{l_b}\right)^2 - (\varphi + 4)\left(\frac{\bar{x}}{l_b}\right) + (1+\varphi)}{(1+\varphi)} \tag{8}$$

$$N_{\theta 3} = \frac{-6\left\{\left(\frac{\bar{x}}{l_b}\right)^2 - \left(\frac{\bar{x}}{l_b}\right)\right\}}{(1+\varphi)l_b} \tag{9}$$

$$N_{\theta 4} = \frac{3\left(\frac{\bar{x}}{l_b}\right)^2 - (2-\varphi)\left(\frac{\bar{x}}{l_b}\right)}{(1+\varphi)} \tag{10}$$

Where φ is the ratio of the beam bending stiffness to shear stiffness and given by $\varphi = \frac{12}{{l_b}^2}\left(\frac{E_b I_b}{\mathrm{K} G A_b}\right)$ where l_b is the length of the beam element, E_b is young's modulus of the beam element, I_b is the second moment of area of the beam element, K is the shear coefficient, G is the shear modulus, A_b is the cross sectional area of the beam element.

The strain energy (V) and the kinetic energy (T) of the beam element is obtained as

$$V^e = \frac{1}{2}\int_0^{l_b} \begin{bmatrix} \frac{\partial \theta^e(x,t)}{\partial x} \\ \frac{\partial w^e(x,t)}{\partial x} + \theta^e(x,t) \end{bmatrix}^T \begin{bmatrix} E_b I_b & 0 \\ 0 & kGA_b \end{bmatrix} \begin{bmatrix} \frac{\partial \theta^e(x,t)}{\partial x} \\ \frac{\partial w^e(x,t)}{\partial x} + \theta^e(x,t) \end{bmatrix} dx \tag{11}$$

$$T^e = \frac{1}{2}\int_0^{l_b} \begin{bmatrix} \frac{\partial w^e(x,t)}{\partial t} \\ \frac{\partial \theta^e(x,t)}{\partial t} \end{bmatrix}^T \begin{bmatrix} \rho_b A_b & 0 \\ 0 & \rho_b I_b \end{bmatrix} \begin{bmatrix} \frac{\partial w^e(x,t)}{\partial t} \\ \frac{\partial \theta^e(x,t)}{\partial t} \end{bmatrix} dx \tag{12}$$

The total work done (W^e) to the external force in the beam is given by

$$W^e = \int_0^{l_b} \begin{bmatrix} w^e(x,t) \\ \theta^e(x,t) \end{bmatrix}^T \begin{bmatrix} F \\ M \end{bmatrix} dx \tag{13}$$

Where F indicates forces and M indicates moment along the length of the beam element.

We get the mass and stiffness matrices of the beam element by substituting the shape functions into the Hamiltonian equation and integrating over the entire length of the beam element which as following:

$$\int_{t_1}^{t_2} \delta\left(T^e - V^e\right) dt + \int_{t_1}^{t_2} \delta W^e dt = 0 \tag{14}$$

$$\left[M^b\right] = \int_0^{l_b} \begin{bmatrix} N_w \\ N_\theta \end{bmatrix}^T \begin{bmatrix} \rho_b A_b & 0 \\ 0 & \rho_b I_b \end{bmatrix} \begin{bmatrix} N_w \\ N_\theta \end{bmatrix} dx \tag{15}$$

$$\left[K^b\right] = \int_0^{l_b} \begin{bmatrix} \frac{\partial}{\partial x} N_\theta \\ N_\theta + \frac{\partial}{\partial x} N_w \end{bmatrix}^T \begin{bmatrix} E_b I_b & 0 \\ 0 & kGA_b \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial x} N_\theta \\ N_\theta + \frac{\partial}{\partial x} N_w \end{bmatrix} dx \tag{16}$$

Now, we obtain the mass matrix and the stiffness matrix of the beam element by substituting the mode shape functions into Eqs. (15), (16) which as following:

$$\left[M^{b}\right]=\frac{\rho_b A_b l_b}{210(1+\varphi)^2}\begin{bmatrix} 70\varphi^2+147\varphi+78 & (35\varphi^2+77\varphi+44)\frac{l_b}{4} & 35\varphi^2+63\varphi+27 & -(35\varphi^2+63\varphi+26)\frac{l_b}{4} \\ (35\varphi^2+77\varphi+44)\frac{l_b}{4} & (7\varphi^2+14\varphi+8)\frac{l_b{}^2}{4} & (35\varphi^2+63\varphi+26)\frac{l_b}{4} & -(7\varphi^2+14\varphi+6)\frac{l_b{}^2}{4} \\ 35\varphi^2+63\varphi+27 & (35\varphi^2+63\varphi+26)\frac{l_b}{4} & 70\varphi^2+147\varphi+78 & -(35\varphi^2+77\varphi+44)\frac{l_b}{4} \\ -(35\varphi^2+63\varphi+26)\frac{l_b}{4} & -(7\varphi^2+14\varphi+8)\frac{l_b{}^2}{4} & -(35\varphi^2+77\varphi+44)\frac{l_b}{4} & (7\varphi^2+14\varphi+8)\frac{l_b{}^2}{4} \end{bmatrix} + \frac{\rho_b I_b}{30(1+\varphi)^2 l_b}\begin{bmatrix} 36 & -(15\varphi-3)l_b & -36 & -(15\varphi-3)l_b \\ -(15\varphi-3)l_b & (10\varphi^2+5\varphi+4)l_b{}^2 & (15\varphi-3)l_b & (5\varphi^2-5\varphi-1)l_b{}^2 \\ -36 & (15\varphi-3)l_b & 36 & (15\varphi-3)l_b \\ -(15\varphi-3)l_b & (5\varphi^2-5\varphi-1)l_b{}^2 & (15\varphi-3)l_b & (10\varphi^2+5\varphi+4)l_b{}^2 \end{bmatrix} \tag{17}$$

$$\left[K^{b}\right]=\frac{E_b I_b}{(1+\varphi)l_b{}^3}\begin{bmatrix} 12 & 6l_b & -12 & 6l_b \\ 6l_b & (4+\varphi)l_b{}^2 & -6l_b & (2-\varphi)l_b{}^2 \\ -12 & -6l_b & 12 & -6l_b \\ 6l_b & (2-\varphi)l_b{}^2 & -6l_b & (4+\varphi)l_b{}^2 \end{bmatrix} \tag{18}$$

2.2. Finite Element modeling of the cracked beam element

According to Dimarogonas [16], the additional strain energy due to existence of crack can be expressed as

$$\prod = \int U dA_c \tag{19}$$

Where U = the strain energy release rate and A_c = the effective cracked area.

$$U=\frac{1}{E}\left[\left(\sum_{n=1}^{2}\kappa_{\text{l}n}\right)^2+\left(\sum_{n=1}^{2}\kappa_{l\,\text{l}n}\right)^2+(1+\nu)\left(\sum_{n=1}^{2}\kappa_{ll\,\text{l}n}\right)^2\right] \tag{20}$$

where $\nu = 0.3$ is Poisson ratio and $\kappa_l, \kappa_{ll}, \kappa_{lll}$ =stress intensity factors for opening, sliding and tearing type cracks respectively. The expressions for stress intensity factors due to load P are given by

$$\kappa_{l1}=\frac{6P_1 l_c}{bh^2}\sqrt{\pi\zeta}F_l\left(\frac{\zeta}{h}\right) \tag{21}$$

$$\kappa_{l2}=\frac{6P_2}{bh^2}\sqrt{\pi\zeta}F_l\left(\frac{\zeta}{h}\right) \tag{22}$$

$$\kappa_{ll1}=\frac{P_2}{bh}\sqrt{\pi\zeta}F_{ll}\left(\frac{\zeta}{h}\right) \tag{23}$$

$$F_l(s)=\sqrt{\frac{\tan(\frac{\pi s}{2})}{\frac{\pi s}{2}}}\left[\frac{0.923+0.199(1-\sin(\frac{\pi s}{2}))^4}{\cos(\frac{\pi s}{2})}\right] \quad (24)$$

$$F_{ll}(s)=\frac{1.122-0.561s+0.085s^2+0.18s^3}{\sqrt{1-s}} \quad (25)$$

Where $s=\zeta/h$

And $F_l(s), F_{ll}(s)$ are correction factors for stress intensive factors. From definition, the elements of the overall additional flexibility matrix C_{ij} can be expressed as

$$C_{ij}=\frac{\partial\delta_i}{\partial P_j}=\frac{\partial^2\Pi_c}{\partial P_i\partial P_j} \quad (26)$$

Substituting i, j (1, 2) values, we get

$$C_{11}=\frac{2\pi}{Eb}\left[\frac{36l_c^{\,2}}{h^2}\int_0^{\zeta/h}xF_l^{\,2}(x)dx+\int_0^{\zeta/h}xF_{ll}^{\,2}(x)dx\right] \quad (27)$$

$$C_{12}=\frac{72\pi l_c}{Ebh^2}\int_0^{\zeta/h}xF_l^2(x)dx=C_{21} \quad (28)$$

$$C_{22}=\frac{72\pi}{Ebh^2}\int_0^{\zeta/h}xF_l^{\,2}(x)dx \quad (29)$$

Now, the overall flexibility matrix C_{ovl} is given by,

$$C_{ovl}=\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix} \quad (30)$$

Flexibility matrix $C_{healthy}$ of the healthy beam element is given by

$$C_{healthy}=\begin{bmatrix}\frac{l_b^{\,3}}{3E_bI_b} & \frac{l_b^{\,2}}{2E_bI_b}\\ \frac{l_b^{\,2}}{2E_bI_b} & \frac{l_b}{E_bI_b}\end{bmatrix} \quad (31)$$

Total flexibility matrix C_{total} of the cracked beam element is given by

$$C_{total}=\begin{bmatrix}\frac{l_b^{\,3}}{3E_bI_b}+C_{11} & \frac{l_b^{\,2}}{2E_bI_b}+C_{12}\\ \frac{l_b^{\,2}}{2E_bI_b}+C_{21} & \frac{l_b}{E_bI_b}+C_{22}\end{bmatrix} \quad (32)$$

Hence the stiffness matrix K_c of a cracked beam element can be obtained as by

$$K_c=\overline{T}C_{total}^{-1}\overline{T}^T \quad (33)$$

Where $\overline{T}$ is the transformation matrix for equilibrium condition.

$$\overline{T}=\begin{bmatrix}-1 & 0\\ l_b & -1\\ 1 & 0\\ 0 & 1\end{bmatrix} \quad (34)$$

2.3. Finite Element Modeling of Piezoelectric Beam Element

The beam elements with the piezoelectric patches are shown in Fig.2. For purposes such as sensing and actuating on the host structure can be used piezoelectric element in flexible structures. The piezoelectric patch characteristics are given in Table 1. The piezoelectric sensor and actuator is also modelled using the Timoshenko beam theory. Similar extraction procedure mass and stiffness matrices for the beam element, we get mass and stiffness matrices for the piezoelectric element which following:

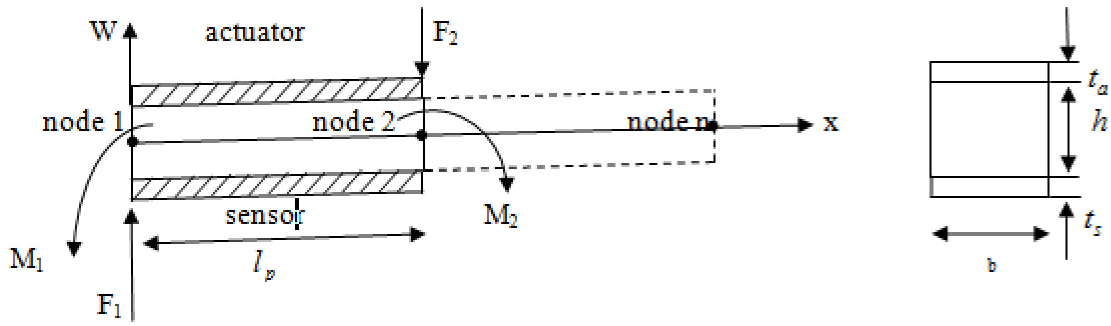


Fig. 2: A piezoelectric beam element

$$[M^p]=\frac{\rho_p A_p l_p}{210(1+\varphi)^2}\begin{bmatrix} 70\varphi^2+147\varphi+78 & (35\varphi^2+77\varphi+44)\frac{l_p}{4} & 35\varphi^2+63\varphi+27 & -(35\varphi^2+63\varphi+26)\frac{l_p}{4} \\ (35\varphi^2+77\varphi+44)\frac{l_p}{4} & (7\varphi^2+14\varphi+8)\frac{l_p^2}{4} & (35\varphi^2+63\varphi+26)\frac{l_p}{4} & -(7\varphi^2+14\varphi+6)\frac{l_p^2}{4} \\ 35\varphi^2+63\varphi+27 & (35\varphi^2+63\varphi+26)\frac{l_p}{4} & 70\varphi^2+147\varphi+78 & -(35\varphi^2+77\varphi+44)\frac{l_p}{4} \\ -(35\varphi^2+63\varphi+26)\frac{l_p}{4} & -(7\varphi^2+14\varphi+8)\frac{l_p^2}{4} & -(35\varphi^2+77\varphi+44)\frac{l_p}{4} & (7\varphi^2+14\varphi+8)\frac{l_p^2}{4} \end{bmatrix} + \frac{\rho_p I_p}{30(1+\varphi)^2 l_b}\begin{bmatrix} 36 & -(15\varphi-3)l_p & -36 & -(15\varphi-3)l_p \\ -(15\varphi-3)l_p & (10\varphi^2+5\varphi+4)l_p^2 & (15\varphi-3)l_p & (5\varphi^2-5\varphi-1)l_p^2 \\ -36 & (15\varphi-3)l_p & 36 & (15\varphi-3)l_p \\ -(15\varphi-3)l_p & (5\varphi^2-5\varphi-1)l_p^2 & (15\varphi-3)l_p & (10\varphi^2+5\varphi+4)l_p^2 \end{bmatrix} \tag{35}$$

$$[K^p]=\frac{E_p I_p}{(1+\varphi)l_p^3}\begin{bmatrix} 12 & 6l_p & -12 & 6l_p \\ 6l_p & (4+\varphi)l_p^2 & -6l_p & (2-\varphi)l_p^2 \\ -12 & -6l_p & 12 & -6l_p \\ 6l_p & (2-\varphi)l_p^2 & -6l_p & (4+\varphi)l_p^2 \end{bmatrix} \tag{36}$$

Where ρ_p is the mass density of the piezoelectric beam element, A_p is the area of the piezoelectric patch, l_p is the length of the piezoelectric beam element, E_p is the modulus of the piezoelectric material, I_p is the moment of inertia of the piezoelectric patch.

TABLE I: Parameters of Beam

Parameters	Beam element	Piezoelectric element
Young's Modulus(Gpa)	E_b =200	E_p =68
Density(kg/m^3)	ρ_b =7860	ρ_p =7700
Thickness(mm)	h =10	t_a, t_s =5
Width(m)	b =0.01	b =0.01
Length(m)	l_b =0.5	l_p =0.1
PZT strain constant(m/V)	——	$d_{31}=125\times10^{-12}$
PZT stress constant(Vm/N)	——	$e_{31}=10.5\times10^{-3}$

The sensor equation is derived from the direct PZT equation which is used to calculate the total charge created by the strain in the structure. The output current of the piezo sensor measures the moment rate of the flexible beam. This current is converted into the open circuit sensor voltage V^s using a signal conditioning device with the gain G_c and applied to an actuator with a suitable gain. Thus, the sensor output voltage $V^s(t)$ is obtained as

$$V^s(t) = G_c e_{31} z b [0 \quad -1 \quad 0 \quad 1]\{\dot{q}\} \tag{37}$$

Where $z = \left(\frac{h}{2} + t_a\right)$

This sensor voltage is given as input to the controller and the output of the controller is the controller gain multiplied by the sensor voltage $V^s(t)$. Thus, the input voltage to the actuator $V^a(t)$,i.e. the control input is given by

$$V^a(t) = Controllergain \times V^s(t) \tag{38}$$

The control force (F_{ctrl}) produced by the actuator that is applied on the beam element is obtained as

$$F_{ctrl} = E_p d_{31} b \bar{z} \int_{l_p} N_\theta dx V^a(t) \tag{39}$$

Where $\bar{z} = \left(\frac{t_a + h}{2}\right)$

2.4. Finite Element modeling of moving load

A beam under a moving load, the force is zero on all nodes except the node that is under force. According to [17] the external force vector takes the following form:

$$\{F^e(t)\} = [F_1(t) F_2(t) F_3(t) F_4(t)]^T = P\{N_w\}^T \tag{40}$$

In order to simulate the moving load, forces and moments to consider as a function of time. Thus,

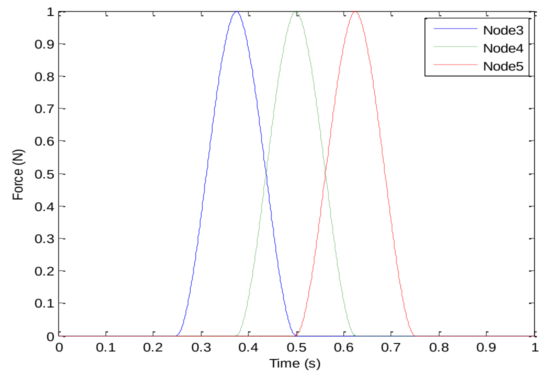
$$t_{\max} = s\Delta t \tag{41}$$

$$[F]^i_{s+1} = [F_{t=0} F_{t=\Delta t} F_{t=2\Delta t} \ldots F_{t=s\Delta t}]_{s+1} \tag{42}$$

$$[M]^i_{s+1} = [M_{t=0} M_{t=\Delta t} M_{t=2\Delta t} \ldots M_{t=s\Delta t}]_{s+1} \tag{43}$$

Where s is time steps, Δt is the total time and (i) is the node number.

So we obtained force and moment functions of time at each node of the beam when it is under a moving load.



In the Fig.3 force versus time for a simply supported beam of length 1 m with 9 nodes.

Fig. 3: Force-time graphs for nodes 3-5

2.5. Equation of Motion

The equation of motion for a Timoshenko cracked beam subjected to a moving load with Piezoelectric patches and neglecting the damping effect of the beam can be written in matrix form as

$$\left[M^*\right]^e \{\ddot{q}\}^e + \left[K^*\right]^e \{q\}^e = \left\{F^t\right\}^e \tag{44}$$

Where $\left[M^*\right]^e$ and $\left[K^*\right]^e$ are the global mass and stiffness matrices of the Timoshenko cracked beam with PZT under a moving load and $\{q\}^e$ is the vector of displacements and slopes and $\left\{F^t\right\}^e$ is the total vector of force which the sum of the vector external force with the vector controlling force obtained. The beam divided into 8 finite elements that piezo-patch and crack placed at positions 2, 4 elements for assembled using the FEM technique. The equation of motion for the Timoshenko cracked beam with piezoelectric patches under a moving load is solved by the Newmark method.

3. Numerical Result

The numerical values of the parameters used in the simulation study are presented in Table 1. In Table 2, natural frequency for each of healthy beam, cracked beam and repaired beam using the Timoshenko and Euler-Bernoulli theories is presented for h=b=0.01m ,a=0.005m and V=10 m/s.

TABLE II. Natural frequencies

Mode no	Euler-Bernoulli (HZ)			Timoshenko (HZ)		
	Healthy	Cracked	Repaired	Healthy	Cracked	Repaired
1	28.66	27.19	28.67	28.65	27.18	28.93
2	112.53	106.76	112.53	112.31	106.52	112.43
3	245.54	232.94	245.54	244.52	231.97	244.69
4	418.76	397.27	418.76	419.89	394.55	416.07

Is shown in Fig.4 displacement the midpoint of each of the beam healthy, cracked and repaired. It is clear in Fig.4 the healthy and repaired beam graphs are approximately coincide. So as expected the maximum displacement of the cracked beam using the piezoelectric patch closer. Is shown in Fig.5, the effect of speed increase moving load of a cracked beam. Is shown in Fig.6, the effect of the piezoelectric patch length on the midpoint displacement of the repaired beam, in which it appears the length of piezoelectric displacement effect is low, generally by increasing the length of the piezoelectric displacement of repaired beams increases.

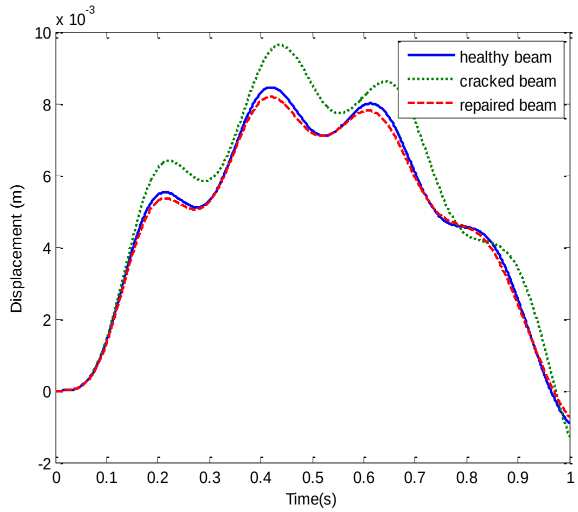


Fig. 4: Displacement versus time at the mid span of beam.

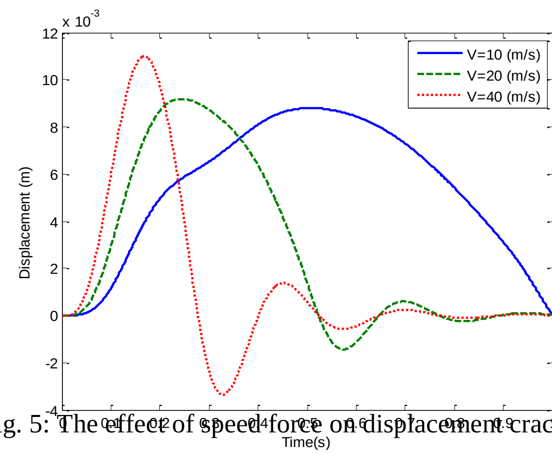


Fig. 5: The effect of speed force on displacement cracked beam.

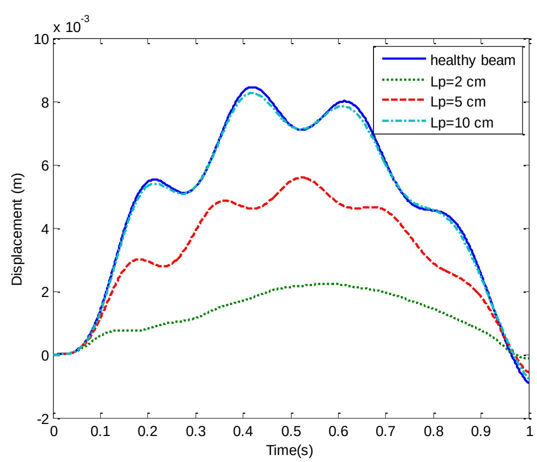


Fig. 6: Effects of piezoelectric length on the repaired beam.

4. Conclusion

The paper introduces a finite element method for the repair of a Timoshenko cracked beam under a moving load using a piezoelectric patch. The beam equations of motion were obtained based on the Timoshenko beam theory. Our criteria for use of piezoelectric patch changes in first frequency cracked beam toward healthy beam. In comparison cracked beam and healthy beam the most displacement is cracked beam where as it reduced using the piezoelectric patch.

5. References

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