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One-Learning-Epoch Optimal LQDT with Input Constraint for the Repetitive Proper System with Unknown Disturbances

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Abstract**:** *In this paper, at first we construct a new iterative learning linear quadratic digital tracker with input constraint for the discrete-time repetitive proper system with an input-to-output direct-feedthrough term, unknown process disturbance, and unknown measurement noise. By the proposed iterative learning control, it can converge in one learning epoch for a desired tracking performance. Subsequently, a numerical simulation is given to demonstrate the effectiveness of the new application of our proposed approach.*

Keywords: *optimal linear quadratic tracker, optimal iterative learning control, discrete-time repetitive system.*

1. Introduction

Iterative learning control considers systems that repetitively perform the same task with a view to sequentially improving accuracy. The specified task is regarded as improving the tracking performance of systems. The objective of iterative learning control (ILC) [1-6] is to use the information from previous executions of the task and do repetitive work by tracking error in attempt to achieve the desired trajectory to minimal error, which has been successfully applied to the real systems, such as industrial robots, wafer scanner, chemical processes and many production machines.

In this paper, a new approach named by the optimal error compensation iterative learning digital tracker (OECILDT) is proposed for the discrete-time proper system with unknown deterministic disturbances. The proposed OECILDT uses the accumulated error vector of each iteration to perform the repetitive task for tracking the desired trajectory. It is an error collection concept that accumulates the errors between output and reference during every iteration, and then uses it to improve the output tracking performance. Basically, our proposed ILC is identical to the optimal ILC [5] for this special case, although both formulations are quite different. However, due to the initialization of the methodology itself, it significantly reduces the learning epochs for the pre-specified tracking performance than that in [5]. This implies that by the well-selected initialization of ILC, it significantly reduces the learning epochs of ILC, which can be referred to our previous work [6]. Besides, the systems considered in [5, 6] and therein are restricted to input constraint-free, disturbance-free and/or strictly proper systems. So no appropriate numerical comparison can be made in this paper.

The paper is organized as follows. A generalized optimal LQDT for discrete-time proper system with known/estimated system disturbances is briefly described in Sec. 2. A new iterative learning LQDT with input constraint for the repetitive discrete-time proper system with a direct-feedthrough term and unknown disturbances is presented in Sec. 3. A numerical simulation is given in Sec. 4 to demonstrate the effectiveness of our proposed approach. Finally, conclusion is given in Sec. 5.

2. A Generalized Optimal Tracker for the Proper System with System Disturbances

A generalized optimal LQDT with pre-specified output and bias of control input trajectories for the linear, controllable and observable discrete-time proper system with known/estimated system disturbances has been presented in [7]. In this section, we briefly describe the generalized LQDT as follows.

Consider the linear discrete-time minimum phase system described by

$$x_d(k+1) = Gx_d(k) + Hu_d(k) + d(k), \tag{1a}$$

$$y_d(k) = Cx_d(k) + Du_d(k) + s(k), \tag{1b}$$

where $x_d(k) \in \Re^n$ is the state vector, $u_d(k) \in \Re^m$ is the control input vector, $y_d(k) \in \Re^p$ is the measured output vector, $m \geq p$, $d(k) \in \Re^n$ is the process disturbance, and $s(k) \in \Re^p$ is the measurement disturbance. The objective is to determine the optimal control sequence $u_d(0),\ u_d(1),\ u_d(2),\ \cdots,\ u_d(N-1)$ that minimizes the following linear quadratic performance index for a finite time process ($0 \leq k \leq N$)

$$J(y_d, u_d) = \frac{1}{2}\left[y_d(N) - r_d(N)\right]^T S\ \left[y_d(N) - r_d(N)\right] + \frac{1}{2}\sum_{k=0}^{N-1}\Big\{\left[y_d(k) - r_d(k)\right]^T Q_d \left[y_d(k) - r_d(k)\right] + \left[u_d(k) - u_d^*(k)\right]^T R_d \left[u_d(k) - u_d^*(k)\right]\Big\}, \tag{2}$$

where Q_d is a $p \times p$ positive definite or positive semi-definite real symmetric matrix, R_d is an $m \times m$ positive definite real symmetric matrix, S is a $p \times p$ positive definite or positive semi-definite real symmetric matrix, $r_d(k)$ is a pre-specified output trajectory, and $u_d^*(k)$ is a pre-specified bias of control input trajectory if it's available.

The minimization problem subjected to equality constraint in (1a) can be solved by adjoining the equality constraint to the quadratic performance index to be minimized by use of Lagrange multiplier [8]. Hence, if the final time $k = N$ goes to infinity, the optimal control vector $u_d(k)$ can be determined as

$$u_d(k) = -K_d x_d(k) + E_d r_d(k) + C_d(k) + C^* u_d^*(k), \tag{3}$$

where

$$K_d = \tilde{R}_d^{-1}\overline{P}_d, \quad E_d = \tilde{R}_d^{-1}\left\{D^T + H^T\left[I_n - (G-HK_d)^T\right]^{-1}(C-DK_d)^T\right\}Q_d,$$
$$C_d(k) = Z_d\, d(k) - E_d\, s(k), \quad C^* = \tilde{R}_d^{-1}\left\{H^T\left[(G-HK_d)^T - I_n\right]^{-1}K_d^T + I_m\right\}R_d,$$

in which

$$Z_d = \tilde{R}_d^{-1}H^T\left\{\left[(G-HK_d)^T - I_n\right]^{-1}(G-HK_d)^T - I_n\right\}P_d,$$
$$\overline{R}_d = R_d + D^T Q_d D,\ N_d = C^T Q_d D,\ \tilde{R}_d = \overline{R}_d + H^T P_d H,\ \overline{P}_d = N_d^T + H^T P_d G,$$

and P_d is the positive definite solution of the following algebraic Riccati equation (ARE)

$$P_d = G^T P_d G + C^T Q_d C - \left(H^T P_d G + N_d^T\right)^T\left[\overline{R}_d + H^T P_d H\right]^{-1}\left(H^T P_d G + N_d^T\right).$$

3. An Optimal Iterative Learning LQDT with Input Constraint for the Discrete-Time Repetitive System with Unknown Process Disturbance and Measurement Noise

Consider the discrete-time repetitive system with unknown deterministic process disturbance and measurement noise described by

$$x_{dj}(k+1) = Gx_{dj}(k) + Hu_{dj}(k) + d(k),\ \ x_{dj}(0) = x_0,\ \text{for}\ j = 0,\ 1,\ 2,\ \cdots, \tag{4a}$$

$$y_{dj}(k) = Cx_{dj}(k) + Du_{dj}(k) + s(k), \tag{4b}$$

where j is the number of learning epochs, $x_d(k) \in \Re^n$ is the state vector, $u_d(k) \in \Re^m$ is the control input vector, $y_d(k) \in \Re^p$ is the measured output vector, $d(k) \in \Re^n$ and $s(k) \in \Re^p$ are the unknown process disturbance and measurement noise, respectively. The objective is to design an optimal iterative learning LQDT with input constraint such that the controlled system in (4) has a desired tracking performance for a given arbitrary reference trajectory $r_d(k)$ with drastic variations.

By observing (3), one can see that if the ratio of Q_d to R_d is high enough, the direct-feedthrough term D has full row rank, and the control input $u_d(k) = -K_d x_d(k) + E_d r_d(k)$, then the system output in (1b) is reduced to

$$\begin{aligned} y_d(k) &= Cx_d(k) + Du_d(k) + s(k) = Cx_d(k) + D\left[-K_d x_d(k) + E_d r_d(k)\right] + s(k) \\ &= \left(C - DK_d\right) x_d(k) + DE_d r_d(k) + s(k) \approx r_d(k) + s(k), \end{aligned} \tag{5}$$

since $\left(C - DK_d\right) \approx 0_{p\times m}$ and $DE_d \approx I_p$. Hence, we have $y_d(k) - r_d(k) \approx s(k)$ and $C_d(k) \approx -E_d s(k) \approx -E_d\left[y_d(k) - r_d(k)\right]$. Based on the observation, since $d(k)$ and $s(k)$ are unknown disturbances, one may imagine that there exists an equivalently undetermined artificial system model

$$x_a(k+1) = Gx_a(k) + Hu_a(k), \tag{6a}$$

$$y_a(k) = Cx_a(k) + Du_a(k) + s_a(k), \tag{6b}$$

where $s_a(k)$ denotes the actual steady-state error signal, to be determined later, between the actual system output $y_{dj}(k)$ and the desired output trajectory $r_d(k)$ as $j \to \infty$. In other words, the whole influence to the repetitive system caused by the unknown disturbance $d(k)$ and noise $s(k)$ may be seen as the influence to the equivalently artificial system caused by the actual steady-state error signal $s_a(k)$. Hence, by applying (3), the optimal control law $u_a(k)$ for the artificial system is given as

$$u_a(k) = -K_a x_a(k) + E_a r_d(k) + C_a(k) + C^* u_a^*(k), \tag{7}$$

where

$$\begin{aligned} &K_a = \tilde{R}_a^{-1}\overline{P}_a, \quad E_a = \tilde{R}_a^{-1}\left\{D^T + H^T\left[I_n - \left(G - HK_a\right)^T\right]^{-1}\left(C - DK_a\right)^T\right\}Q_a, \\ &C_a(k) = -E_a s_a(k), \quad C^* = \tilde{R}_a^{-1}\left\{H^T\left[\left(G - HK_a\right)^T - I_n\right]^{-1}K_a^T + I_m\right\}R_a, \end{aligned}$$

in which

$$\overline{R}_a = R_a + D^T Q_a D, \;\; N_a = C^T Q_a D, \;\; \tilde{R}_a = \overline{R}_a + H^T P_a H, \;\; \overline{P}_a = N_a^T + H^T P_a G,$$

and P_a is the positive definite solution of the following algebraic Riccati equation

$$P_a = G^T P_a G + C^T Q_a C - \left(H^T P_a G + N_a^T\right)^T\left[\overline{R}_a + H^T P_a H\right]^{-1}\left(H^T P_a G + N_a^T\right).$$

To determine the actual steady-state error signal $s_a(k)$, let us modify it as

$$s_{aj}(k) = \sum_{i=0}^{j} e_{i-1}(k) = \sum_{i=0}^{j}\left[y_{d(i-1)}(k) - r_d(k)\right], \tag{8}$$

where $e_{-1}(k) = 0,$ for the repetitive system in (4). Thus, the OECILDT for the repetitive system is given by

$$u_{dj}(k) = -K_d x_{dj}(k) + E_d r_d(k) + C_{aj}(k) + C^* u_d^*(k)\ , \tag{9}$$

where

$$\begin{aligned} &K_d = \tilde{R}_d^{-1}\overline{P}_d, \quad E_d = \tilde{R}_d^{-1}\left\{D^T + H^T\left[I_n - \left(G - HK_d\right)^T\right]^{-1}\left(C - DK_d\right)^T\right\}Q_d, \\ &C_{aj}(k) = -E_d s_{aj}(k), \quad C^* = \tilde{R}_d^{-1}\left\{H^T\left[\left(G - HK_d\right)^T - I_n\right]^{-1}K_d^T + I_m\right\}R_d, \end{aligned}$$

in which

$$\overline{R}_d = R_d + D^T Q_d D, \;\; N_d = C^T Q_d D, \;\; \tilde{R}_d = \overline{R}_d + H^T P_d H, \;\; \overline{P}_d = N_d^T + H^T P_d G,$$

and P_d is the positive definite solution of the following algebraic Riccati equation

$$P_d = G^T P_d G + C^T Q_d C - \left(H^T P_d G + N_d^T\right)^T \left[\overline{R}_d + H^T P_d H\right]^{-1} \left(H^T P_d G + N_d^T\right).$$

The architecture of the optimal error compensation iterative learning control (OECILC) for the repetitive system is depicted in Fig. 1.

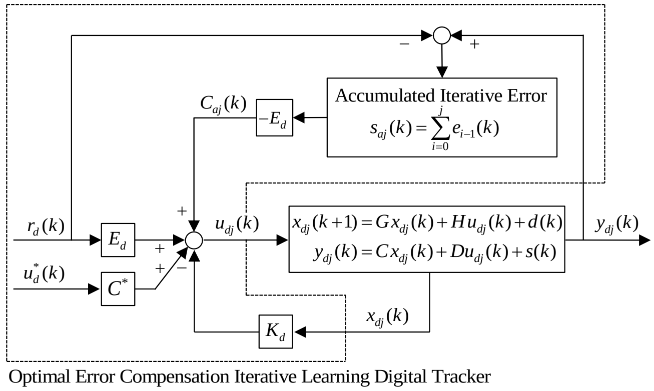


Fig. 1: Optimal error compensation iterative learning control for the repetitive system.

Lemma 1: If the ratio of Q_d to R_d tends to infinity and the direct-feedthrough term D has full row rank, then the system output in (4) is reduced to $y_{dj}(k) \approx r_d(k)$ as $j \geq 1$. ◇

Proof: Let $Q_d = \mu I_p$, $R_d = I_m$, and $\mu \to \infty$. Then, one has $K_d \approx \left(D^T Q_d D\right)^{\dagger} \left(D^T Q_d C\right)$. Since $\text{rank}(D) = p$, we can obtain $(C - DK_d) \approx 0_{p\times n}$. Thus, the forward gain matrix is reduced to

$$E_d \approx \left(D^T Q_d D\right)^{\dagger} D^T Q_d, \tag{10}$$

which implies that $DE_d \approx I_p$ since $\text{rank}(D) = p$. By using the above results, substituting (9) into (4b), and setting $u_d^*(k) = 0$, one has

$$\begin{aligned} y_{dj}(k) &= Cx_{dj}(k) + Du_{dj}(k) + s(k) = Cx_{dj}(k) + D\left[-K_d x_{dj}(k) + E_d r_d(k) + C_{aj}(k)\right] + s(k) \\ &= (C - DK_d) x_{dj}(k) + DE_d r_d(k) + -DE_d s_{aj}(k) + s(k) \approx r_d(k) + \left[s(k) - s_{aj}(k)\right]. \end{aligned} \tag{11}$$

Then, since $s_{aj}(k) = \sum_{i=0}^{j} e_{i-1}(k) = \sum_{i=0}^{j} \left[y_{d(i-1)}(k) - r_d(k)\right]$, we have

$$\begin{aligned} & j = 0 : s_{a0}(k) = e_{-1}(k) = 0, \\ & \quad \Rightarrow\ y_{d0}(k) \approx r_d(k) + \left[s(k) - s_{a0}(k)\right] = r_d(k) + s(k), \\ & j = 1 : s_{a1}(k) = e_{-1}(k) + e_0(k) = 0 + \left[y_{d0}(k) - r_d(k)\right] \approx s(k), \\ & \quad \Rightarrow\ y_{d1}(k) \approx r_d(k) + \left[s(k) - s_{a1}(k)\right] \approx r_d(k), \\ & j = 2 : s_{a2}(k) = e_{-1}(k) + e_0(k) + e_1(k) \approx 0 + s(k) + \left[y_{d1}(k) - r_d(k)\right] \approx s(k), \\ & \quad \Rightarrow\ y_{d2}(k) \approx r_d(k) + \left[s(k) - s_{a2}(k)\right] \approx r_d(k), \\ & \quad \vdots \end{aligned}$$

which implies that $s_{aj}(k) \approx s(k)$ and $y_{dj}(k) \approx r_d(k)$ as $j \geq 1$. □

Hence, the strength of this control scheme is that it can converge in one learning epoch. Moreover, for $j = 0,\ 1,\ 2,\ \cdots,$ one has

$$\begin{aligned} & u_{d0}(k) = -K_d x_{d0}(k) + E_d r_d(k) + C^* u_d^*(k), \\ & u_{d1}(k) = -K_d x_{d1}(k) + E_d r_d(k) - E_d e_0(k) + C^* u_d^*(k) \Rightarrow \Delta u_{d1}(k) = u_{d1}(k) - u_{d0}(k) = -K_d\left[x_{d1}(k) - x_{d0}(k)\right] - E_d e_0(k), \\ & u_{d2}(k) = -K_d x_{d2}(k) + E_d r_d(k) - E_d\left[e_0(k) + e_1(k)\right] + C^* u_d^*(k) \Rightarrow \Delta u_{d2}(k) = u_{d2}(k) - u_{d1}(k) = -K_d\left[x_{d2}(k) - x_{d1}(k)\right] - E_d e_1(k), \end{aligned}$$

$$\vdots$$

$$u_{dj}(k) = -K_d x_{dj}(k) + E_d r_d(k) - E_d\left[\sum_{i=0}^{j} e_{i-1}(k)\right] + C^* u_d^*(k) \Rightarrow \Delta u_{dj}(k) = u_{dj}(k) - u_{d(j-1)}(k) = -K_d\left[x_{dj}(k) - x_{d(j-1)}(k)\right] - E_d e_{j-1}(k),$$

which implies that

$$u_{dj}(k) = u_{d(j-1)}(k) + \Delta u_{dj}(k) = u_{d(j-1)}(k) + \left\{-K_d\left[x_{dj}(k) - x_{d(j-1)}(k)\right] - E_d e_{j-1}(k)\right\}. \quad (12)$$

An interesting application of the generalized optimal LQDT design for the input-constraint problem has been studied in this work, where $u_d^*(k)$ is re-set as the pre-specified control input saturation bound, whenever the pre-determined control input exceeds the practical saturation bound at any time instant. Otherwise, it is set as the default circumstance, i.e. the bias of control input trajectory.

Here, we would like to point out that appropriately tuning the weighting matrices $\{Q_d(k), R_d(k)\}$ incorporated with the above-mentioned scheme also provides another degree-of-freedom to achieve an even better performance.

4. An Illustrative Example

Consider the repetitive minimum phase system in (4) with unknown disturbances $d(k)$ and $s(k)$, where

$$G = \begin{bmatrix} 0.9048 & 0.0000 & 0.0000 & -0.0464 \\ 0.0000 & 0.7512 & -0.0238 & -0.0476 \\ 0.0000 & 0.0000 & 0.9112 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & -0.8712 \end{bmatrix}, \quad H = \begin{bmatrix} 0.0505 & -0.3196 & -1.3966 \\ -1.4935 & -0.5775 & -1.1478 \\ 1.4286 & -0.1346 & 0.1959 \\ -0.2338 & -1.4708 & -0.5057 \end{bmatrix},$$

$$C = \begin{bmatrix} 1.2061 & 0.9635 & -1.0511 & 0.2557 \\ -1.1513 & -0.3076 & -0.7540 & 0.0205 \end{bmatrix}, \quad D = \begin{bmatrix} 0.5203 & 10.1414 & -18.5353 \\ 8.7760 & -0.8435 & -10.9682 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 1.1 & -0.8 & 2.5 & -1.3 \end{bmatrix}^T,$$

with the sampling time $T_s = 0.1$ sec. The stochastic measurement disturbance $s(k) = \begin{bmatrix} s_1(k) & s_2(k) \end{bmatrix}^T$ and deterministic system disturbance $d(k) = \begin{bmatrix} d_1(k) & d_2(k) & d_3(k) d_4(k) \end{bmatrix}^T$ are created by $s_1(k) = N(3,5)$, $s_2(k) = N(2,4)$, $d_1(k) = 3.2\sin(5\pi k)$, $d_2(k) = 4.3(k-1)^2$, $d_3(k) = 2.5\cos(3\pi k)$, and $d_4(k) = \begin{cases} -2.3 & 0.0 \le k < 1.5\,\text{sec} \\ 1.7, & 1.5 \le k < 2.5\,\text{sec} \\ 3.5, & 2.5 \le k \le 3.0\,\text{sec} \end{cases}$, where $N(\alpha, \beta)$ denotes a normal distribution white noise sequence with mean α and standard deviation β.

Case 1. OECILC without input-constraint for the repetitive system with a direct-feedthrough term and unknown deterministic system and measurement disturbances

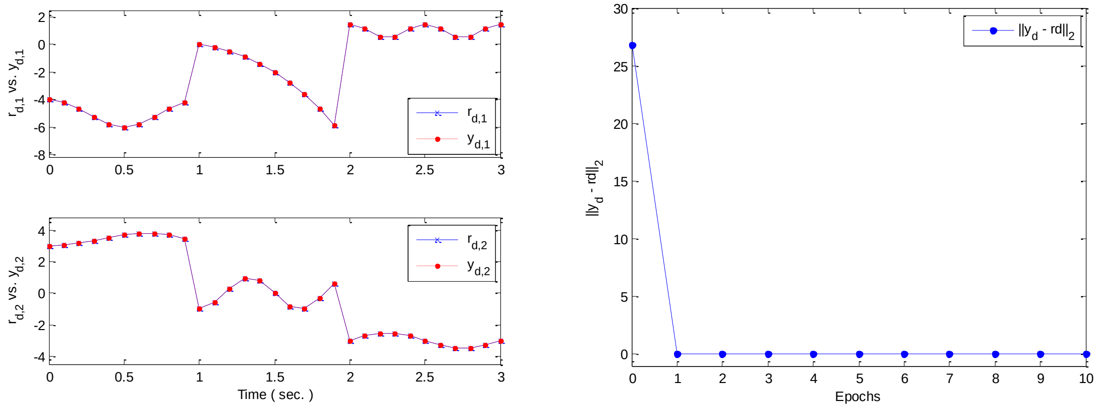
It is required to determine an iterative learning optimal digital tracker for the input-constraint-free case so that the controlled system demonstrates a good tracking performance for the reference $r_d(t) = \begin{bmatrix} r_{d,1}(t) & r_{d,2}(t) \end{bmatrix}^T$, where

$$r_{d,1}(k) = \begin{cases} \cos(2\pi k) - 5, & 0 \le k < 1\,\text{sec} \\ 1.8\,k^2(1-k), & 0 \le k < 1\,\text{sec} \\ 0.5\cos(4\pi k) + 1, & \text{otherwise.} \end{cases}, \quad r_{d,2}(k) = \begin{cases} 5.5\,k^2(1-k) + 3, & 0 \le k < 1\,\text{sec} \\ \cos(3\pi k), & 0 \le k < 1\,\text{sec} \\ 0.5\sin(2\pi k) - 3, & \text{otherwise.} \end{cases}.$$

The proposed OECILC (8)-(9) is then determined, where

$$K_d = \begin{bmatrix} -0.2729 & -0.0824 & -0.0962 & 0.0003 \\ -0.0652 & 0.0262 & -0.0997 & 0.0195 \\ -0.1084 & -0.0399 & -0.0005 & -0.0031 \end{bmatrix}, \quad E_d = \begin{bmatrix} 0.0092 & 0.3567 \\ 0.0978 & 0.2952 \\ -0.0002 & 0.1715 \end{bmatrix}, \quad C_{aj}(k) = -E_d \sum_{i=0}^{j} e_{i-1}(k),$$

for $Q_d = 10^6 I_3$. and $R_d = I_3$. The tracking performance of the first learning epoch is shown in Fig. 2(a), and the learning curve is shown in Fig. 2(b), which demonstrates a satisfied tracking performance as expected.



(a) (b)

Fig. 2. OECILC-based controlled system without input-constraint at the first learning epoch: (a) tracking responses, (b) learning curve.

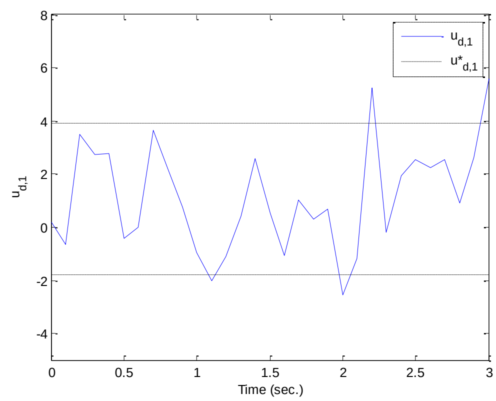
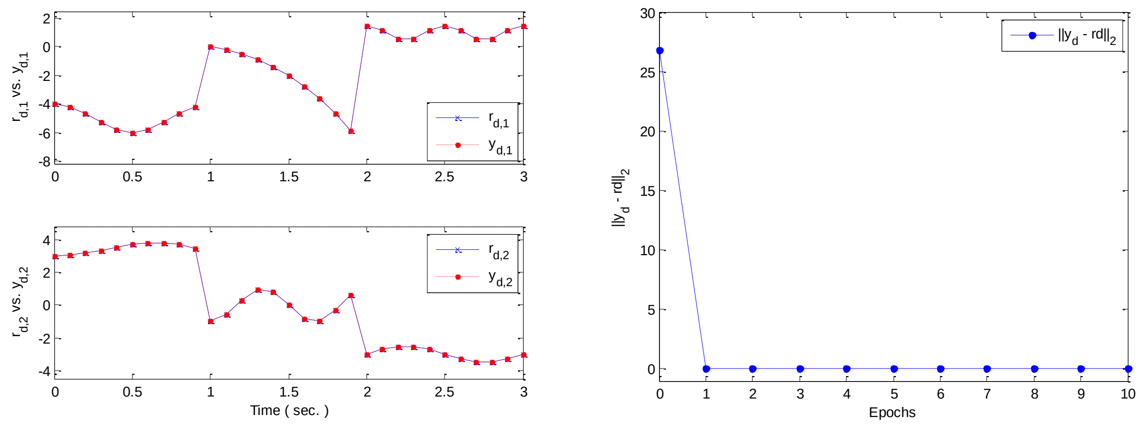


Fig. 3. Control input of the OECILC-based controlled system without input-constraint for the repetitive system at the first learning epoch: $u_{d,1}(k)$ (shown by parts).

Case 2. OECILC with input-constraint for the repetitive system with a direct-feedthrough term, unknown deterministic system and measurement disturbances

The objective of the classical tracking problem is to design an appropriate control law $u_d(k)$ with input constraint so that the system output $y_d(k)$ can well track the pre-specified output target trajectory $r_d(k)$ as possible. In Sec. 2, a tracking problem is presented, so that a trade-off between the output tracking for a pre-specified output target trajectory $r_d(k)$ and the control input tracking for a pre-specified bias of input target trajectory $u_d^*(k)$, in terms of each corresponding weighting matrices Q_d and R_d, is presented. A new application of this problem on the input-constrained digital tracker design is to be given in the following.

Let the discrete-time system have the above-mentioned disturbances. Based on the proposed input-saturation-free LQDT for $Q_d = 10^6 I_2$ and $R_d = I_3$, the tracking performance is quite satisfactory, while the proposed tracker-based control inputs have some acute responses for some time instants (see Fig. 3). Assume it is desired to narrow down the control-input magnitudes at those time instants with unexpected acute responses, and let $u_d^*(k)$ specify the desired time response of the control input. Here, based on the upper and lower bounds of the input constraint-free case $u_{d,sat} = \begin{bmatrix} u_{d,1}^{\min} & u_{d,1}^{\max} \\ u_{d,2}^{\min} & u_{d,2}^{\max} \\ u_{d,3}^{\min} & u_{d,3}^{\max} \end{bmatrix} = \begin{bmatrix} -2.5594 & 5.5998 \\ -3.2688 & 3.1211 \\ -1.5003 & 2.1693 \end{bmatrix}$, the upper and lower bounds of $u_d^*(k)$ is pre-specified as $u^*_{d,sat} = 0.7 \times u_{d,sat} = \begin{bmatrix} -1.7916 & 3.9199 \\ -2.2881 & 2.1848 \\ -1.0502 & 1.5185 \end{bmatrix}$. Whenever the components of $u_d(k)$ exceed the upper or lower bound of $u^*_{d,sat}$, the corresponding components of $u^*_d(k)$ are specified as the pre-specified upper or lower bound; otherwise, it is set to be zero (i.e. the pre-specified bias of control input, for this example). The closed-loop responses of the optimal iterative learning LQDT with input constraint for the repetitive system are shown in Figs. 4 and 5. Fig. 4 shows that both tracking performance and learning curve are well-performed. Fig. 5 shows those acute control inputs are restricted to the pre specified upper and lower bounds for this illustrative example.



(a) (b)

Fig. 4. OECILC-based controlled system with input-constraint at the first learning epoch: (a) tracking responses, (b) learning curve.

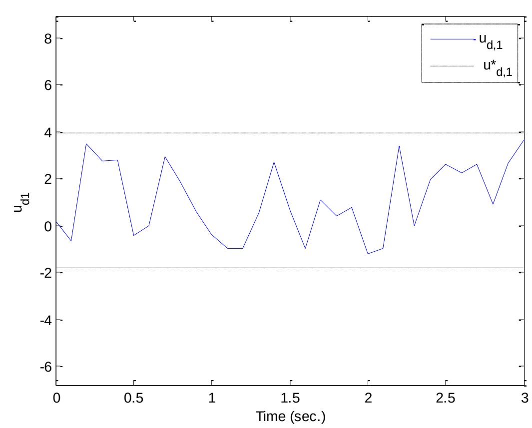


Fig. 5. Control input of the OECILC-based controlled system with input-constraint for the repetitive system at the first learning epoch: $u_{d,1}(k)$ (shown by parts).

5. Conclusion

This paper has developed a new iterative learning LQDT with input constraint for the repetitive discrete-time proper system with a direct-feedthrough term and unknown disturbances has been presented. Theoretical analysis and simulation experience show that the proposed ILC could significantly reduce the learning epochs for the pre-specified tracking performance.

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