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Formulations of Shell Finite Elements via a Hierarchical Approach

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***Abstract*:** *This paper address the formulation of a family of shell finite elements developed by means of a unified formulation. Thanks to this formulation the through-the-thickness kinematics can be freely enriched resulting in hierarchical elements able to yield an accurate response of the structure. The number of nodes per element is also a free parameter. A MITC formulation is used to overcome locking. The weak form of the governing equations is derived in a curvilinear reference system.*

Keywords: *Shell structures, composite materials, piezo-electricity, finite element method, unified formulation.*

1. Introduction

Shell structures are capable of transmitting surface loads by “membrane” stresses. Thanks to this feature, shell structures, under the same loadings, are more rigid and economical than plates. Composite shell structures are used as primary structural components due to their high value of strength- and stiffness-to-weight ratios. In nowadays applications, shells’ thickness is comparable to the radii of curvature. Furthermore, localised loadings may be also present. By means of piezo-electric layers, these structures have been made “smart”, that is, capable of adapt to external stimuli. In such cases, three-dimensional stress states occur locally or, even, globally and classical theories, based on Love’s or Mindlin’s kinematic assumptions, may not yield a correct prediction of displacement and stress fields. For this reason, refined higher-order models are required for an accurate and effective design of smart curved and layered structures.

This work addresses the formulation of a family of structural finite elements for the analysis of piezo-electric composite shell structures. A unified and comprehensive manner to formulate two-dimensional shell models is herein adopted. This approach is known as Carrera's Unified Formulation (CUF) [1-5]. Thanks to the assumption of a unified notation, governing equations that are common to all the CUF models are easily obtained. CUF allows formulating several two-dimensional models on the basis of the choice of the a-priori main unknowns, the approximation level, the through-the-thickness polynomial approximation order. Models that account for the transverse normal and shear deformability, the continuity of the transverse stress components and the zig-zag variation along the thickness of displacement and transverse normal stresses can be formulated straightforwardly. The governing equations are derived in the general case of doubly curved shells. CUF models are capable of predicting the displacement and stress fields as accurately as desired.

2. Preliminaries

A shell is a three-dimensional body bounded by two closely spaced curved surfaces, see Fig. 1. The surface placed midway between the two enveloping ones is called reference surface W_0. Two curvilinear and orthogonal axes (a, b) are identified over the reference surface. The distance between the two surfaces measured along the normal to the reference surface is the shell thickness and it is small when compared to any other dimension laying on the reference surface. The thickness identifies the third axis z that completes the curvilinear reference system (a, b, z) used for deriving the proposed family of finite elements.

Shell elements with constant thickness h and curvature radii R_a and R_b along the a and b directions are considered. The latter hypothesis implies that the coefficients of the first fundamental form of the reference surface are equal to the unit. The shell parametric coefficients are addressed by H_a and H_b. The displacment field id identified by its three components u_a, u_b and u_z in the considered reference system.

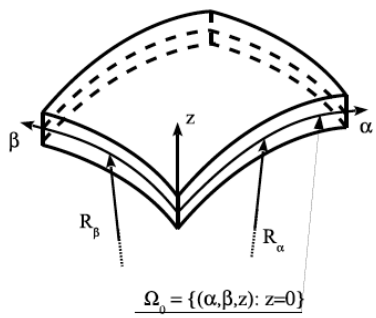

Fig. 1: Laminated shell geometry and notation.

The strain-displacement relations of three-dimensional theory of elasticity in orthogonal curvilinear coordinates is assumed. The strain tensor e in divided into the components laying on the reference surface (e_p) and those normal to it (e_n). According to this arrangement, the matrix form of thre geometrical relations reads:

$$\begin{aligned} e_p &= \left(D_p + A_p\right) u \\ e_n &= \left(D_{np} + D_{nz} - A_n\right) u \end{aligned} \tag{1}$$

Under the hypothesis of linear elastic materials, the material constitutive relations are:

$$\begin{aligned} s_p &= C_{pp} e_p + C_{pn} e_n \\ s_n &= C_{np} e_p + C_{nn} e_n \end{aligned} \tag{2}$$

3. Hierarchical Shell Elements Kinematics

In the framework of the proposed unified finite element modelling, the variation of each displacement component versus the spatial coordinates is:

$$u(a, b, z) = F_t(z) N_i^u(a, b) q_{ti} \quad t = 0, 1, \ldots, N \; i = 1, 2, \ldots, N_n \tag{3}$$

According to Einstein's notation, a repeated index is a dummy index that, unless otherwise stated, stands for summation. This notation is extensively used through the paper and it allows deriving the problem's governing equations in terms of a single "fundamental nucleus" regardless the approximation order over the thickness (N), the number of nodes per element (N_n) over the reference surface. The actual governing equations due to fixed cross-section approximation order and number of nodes per element are obtained straightforwardly via summation of the nucleus corresponding to each term of the expansion. In this sense, N and N_n are free parameters in the formulation. q_{ti} is the nucleus of the element nodal unknown vector. $F(z)_t$ are the a-priori approximating functions over the shell thickness. The dummy index ranges over the number of cross-section approximation terms. N_i^u are the Lagrangian nine nodes finite element shape functions for the displacements. They approximate the displacements over the reference surface in a C^0 sense. Shear and membrane locking are contrasted by means of the Mixed Interpolation of Tensorial Components (MITC), see Bathe and Dvorkin [6].

4. Elements Formulation

The stiffness matrix are derived by means of the principle of virtual displacement written at element level in the orthogonal curvilinear reference system. The virtual variation of the strain energy reads:

$$\delta L_i = \int_V \left(\delta \varepsilon_p^T \sigma_p + \delta \varepsilon_n^T \sigma_n \right) dV \tag{4}$$

By replacing the previour equations, Eq.4 reads:

$$\delta L_i = \delta q_{sj} K^{st\,ji} q_{ti} \tag{5}$$

where $K^{st\,ji}$ is the fundamental nuclueus of the stiffness matrix. The explicit form of its components are not here reportred for the sake of brevity. They can be found in Cinefra and Carrera [3].

5. Numerical Results

A circular cylindrical shell pinched by diametrically opposite loads is considered. The material parameters are: $E_L = 25E_T$, $G_{LT} = 0.5E_T$, $G_{TT} = 0.2E_T$ and $\nu_{LT} = \nu_{TT} = 0.25$. The laminaiton is $[90/0/90]$ versus the α direction. The case under investigation is presented in Fig.2.

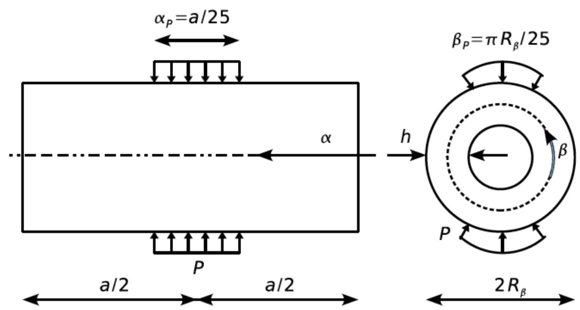

Fig. 2: cylindrical shell pinched by diametrically opposite loads.

The dimensional stranverse displacement $u_z \frac{E_L h}{P}$ for $R_b / h = 100$ is presented in Table 1.

TABLE I: Dimensional stranverse displacement, $R_b / h = 100$.

2z/h	-1	0	1
LD4	−663.1	−663.2	−663.0
LD3	−663.1	−663.2	−663.0
LD2	−663.1	−663.2	−662.9
ED3	−663.0	−663.1	−662.9
FSDT	−657.6	−657.6	−657.6

Where "LD" stands for a layer-wise model base on displacements, "ED" for an equivalent single layer model and FSDT for first order shear deformation theory. The number after the acronym stands for the throught-the-thickness approximation order. A good convergence is observed when the model is hierarchically refined. The case $R_b / h = 10$ is presented in Table 2.

TABLE II: dimensional stranverse displacement, $R_b / h = 10$.

2z/h	-1	0	1
LD4	−93.41	−94.92	−98.87
LD3	−93.41	−94.92	−98.87
LD2	−92.92	−94.44	−98.37
ED3	−92.93	−94.54	−98.77
FSDT	−86.25	−86.25	−86.25

Also in this case, a good converge is observed. FSDT is less accurate since the structure is thick.

6. Conclusions

A family of shell finite elements has been derived to investigate composite shell structures. The derivation is obtained in a unified manner that allows to enrich the kinematics at will so that accurate results, as shown by the numerical investigations, can be obtained. The procedure is also attractive when considering that several degrees of freedom can be saved when comparing to a three-dimensional finite element solution.

7. Acknowledgements

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