

Stochastic Simulation of Maximum Structural Response with Random Damping: Application to the Algerian Seismic Design Code

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Abstract: We present in this paper, the main results of a numerical investigation of the effect of critical damping uncertainties on the maximum response of systems with random damping. For this purpose, the excitation is defined by a specific target response spectrum. The critical damping coefficient is distributed according to the Lognormal probability density function with known mean and standard deviation. The Monte Carlo method is used to perform a second order analysis of the simulated response spectrum amplitudes. The applicability of the proposed methodology is illustrated using several target response spectra corresponding to real accelerogram records and design response spectra of The Algerian Earthquake Resistant Regulations (RPA 2003) associated respectively, with each of the four soil types: rock, firm, soft and very soft. The simulation results are then presented in terms of variations of the amplitudes of response spectra to illustrate quantitatively the effect of the uncertainty of the damping on the maximum structural response and conclusions of engineering importance are given.

Keywords: Algerian Earthquake Resistant Regulations, Response Spectrum, Uncertain damping, Monte Carlo simulation, Accelerograms.

1. Introduction

In order to reliably estimate the seismic response of a building, it is essential to quantify not only the effect of the stochastic nature of the seismic motion on the structural response, but also the uncertainty of the dynamic parameters [1, 2 and 3]. These uncertainties, mainly caused by the variation of material properties and approximations in the estimation of parameters of the mathematical model used, may introduce, for a given structure, a significant variation in reliability and response, and therefore it is often desirable to consider their effects in the analysis of its behavior and design.

The objective of this study is to estimate the effects of damping uncertainties on the dynamic response of structures. Uncertainties are treated by considering the damping as a Lognormal random variable with characteristics obtained on the basis of statistical treatment of a wide range of structures and structural systems. The seismic responses are simulated using the Monte Carlo technique. The applicability of the proposed methodology is illustrated using the target response spectrum corresponding to a record of the Parkfield Earthquake of June 27, 1966 and design response spectra of the Algerian Earthquake Regulations (RPA 2003) respectively for four soil types: rock, firm, soft and very soft. [4]. The simulation results are then presented for response amplitudes lying in intervals at 68% confidence level to illustrate quantitatively the effect of damping uncertainty on maximum structural response and the conclusions of its importance for engineers are given.

First, the mathematical principles related to stability of probabilistic estimates of Monte Carlo method are presented and applied to our study case for the selection of an optimal sampling. This optimal number of simulations is then used in a second step, to analyze the influence of changes of damping values on those of

simulated spectral amplitudes. The analysis is done initially on the spectral amplitude variations obtained for a fixed value of the coefficient of variation of the damping and thereafter on the influence of three different values of the coefficient of variation on the maximum structural responses. Results of some importance for engineers are obtained and their use can be extended to improve current seismic regulations.

2. Simulation Technique

In this work, we use the Monte Carlo method to estimate the value $S(\xi, f)$ of the response spectrum associated to a structure of fundamental frequency f and uncertain damping factor ξ . We assume that the values ξ_i ($i = 1$ to n) of the random variable input ξ are independent and identically distributed, and in our case we choose to make them follow the Log-normal distribution. Under this assumption, the random values of the sample $S(\xi_i, f)$ of size n (i. e. $i = 1$ to n) of corresponding responses are also independent and identically distributed and moreover, by virtue of the law of large numbers, the characteristics of the random sample approach even more statistical characteristics of the population as the sample size n increases. To assess the convergence of this estimate, we use as limit state function $G(m_{S(\xi, f)})$ the probability $P = \text{Prob}(m_{S(\xi, f)} \in [(\mu_{S(\xi, f)} - \sigma(m_{S(\xi, f)})) - (\mu_{S(\xi, f)} + \sigma(m_{S(\xi, f)}))]$, where $m_{S(\xi, f)}$ is the estimation of the mean $\mu_{S(\xi, f)}$ of the population, obtained for a set of 100 samples of size n and $\sigma(m_{S(\xi, f)})$ is the standard - deviation of these estimates. By virtue of the central limit theorem, the variable $m_{S(f)}$ follows the normal distribution with expectation $\mu_{S(f)}$ and variance $\sigma(m_{S(f)})/\sqrt{n}$. We see that when n is large the probability P approaches the value 0.68 while the variance $\sigma(m_{S(f)})/\sqrt{n}$ tends to zero.

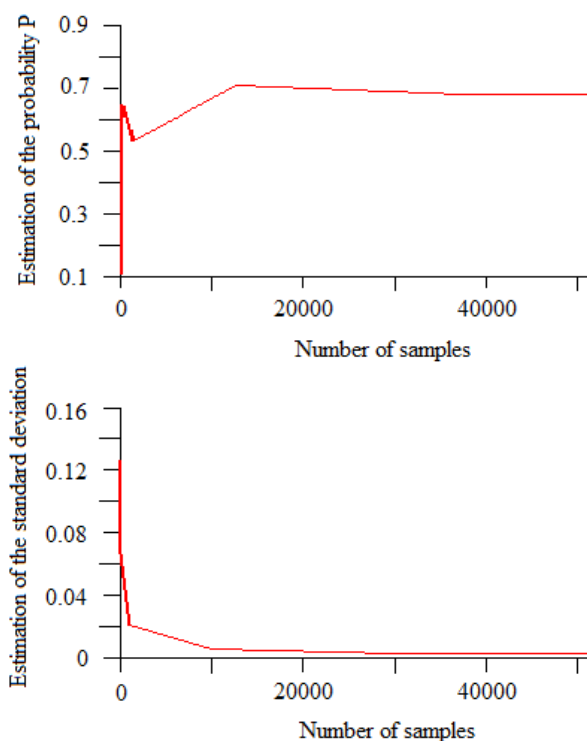


Fig. 1: Convergence of the probability P as a function of the number of samples

Figure 1 above shows the convergence of the probability P as a function of the number of samples n . The stability of the simulation was obtained from $n = 30000$ which is the value that we used to simulate response spectra values in the present study.

3. Numerical Results and Discussion

3.1 Statistical characterization of damping

The selection of an appropriate value of damping is subject to controversy in the practice of design of structures. Evaluation of damping in completed structures was undertaken by several investigators. The

information provided by complete empirical experiments was assembled [5] and has provided a range of information for different levels of response amplitudes and large classes of structural systems and sizes of buildings. This study has shown that Gamma and lognormal distributions provide the best fit to the variations of damping, which have a coefficient of variation (COV) $C_{\xi} = \sigma_{\xi}/\mu_{\xi}$ whose values are contained in the interval [33% - 87%].

3.2 Sensitivity of the Response to the Damping Uncertainty Value

The simulation procedure described above was applied to generate response spectra compatible with those corresponding to the acceleration records of Parkfield Earthquake on 27 June 1966 and with design spectra associated to the Algerian Earthquake Regulations (RPA 2003) for each of four soil types: rock, firm, soft and very soft. A fixed value of the critical damping $\mu_{\xi} = 5\%$, regarded as mean value, was considered for the calculation of sets of response spectra characterized by random damping values with COV value $C_{\xi} = 40\%$ and following the Lognormal probability density function with standard deviation $\Delta\xi = \sigma_{\xi} = C_{\xi} \times \mu_{\xi} = 0.02$.

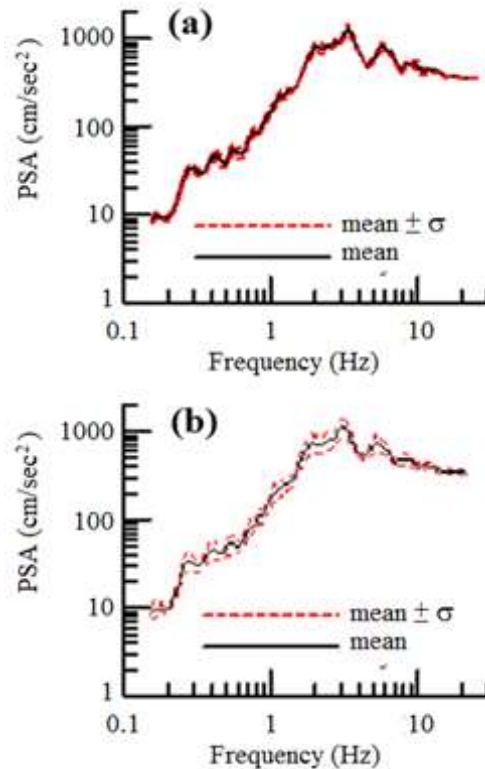


Fig. 2: Mean spectrum ± 1 standard - deviation : a) Monte Carlo simulation b)Analytical approach.

Before starting the analysis of the results, we firstly validate the stochastic simulation procedure above by comparing it to an analytical procedure [6]. Lacking space, we only present in Fig. 2 the results obtained by applying both procedures in response spectrum associated with only a single record of the Parkfield Earthquake. Note the variations of the response spectra simulated by the Monte Carlo method Fig. 2a are smaller than the analytical method variability spectra Fig. 2b.

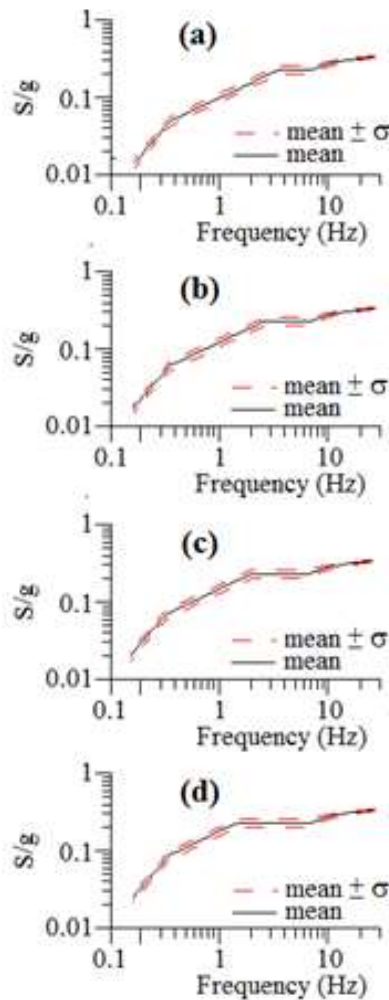


Fig. 3: RPA spectrum mean \pm 1 standard - deviation for different soil types: a) Rock site (S1), b) Firm site (S2), c) Soft site (S3), d) Very soft site (S4).

The Figure 3 shows the results obtained by considering the RPA 2003 design spectra associated with each of the four considered soil types (S1 to S4) and constructions of high importance (category 1B) with a quality factor $Q = 1.2$ and a coefficient of performance for the structure $R = 5$, located in a high seismic zone (zone 3), characterized by an acceleration coefficient $A = 0.3$ for the category concerned structures.

The results show a low frequency range [$< 3\text{Hz}$] where the response of the oscillator is controlled mainly by the displacement of the support (flexible structure) because the mass does not move in the absolute axes and there is a strict equality between maximum displacements (relative for the mass, absolute for the support), regardless to the damping. Another frequency range [$> 8\text{Hz}$], where the response of the oscillator is controlled mainly by the acceleration of the support, in which the influence of damping variations is least. The largest fluctuations are located in the intermediate frequency range [$3\text{Hz} - 8\text{Hz}$] where the values of the spectral response have randomly distributed extrema.

TABLE I : Cov Of Psa Amplitudes :Statistical Characteristics (Parkfield Earthquake Of June 27, 1966)

Mean value of COV : μ_{Crep}	7.70%
Standard-deviation value of COV : σ_{Crep}	4.16%
Range of values of COV $\text{C}_{\text{rep}}(f)$	0.13% - 15.39%

The characteristic values of the coefficient of variation $C_{rep}(f)$ for these extrema are shown in Table I for the PSA associated to the real seismic record and in Table II for design acceleration response spectra required by RPA 2003 regulations. It is seen that the variation range of $C_{rep}(f)$ for the real PSA is between 0.13% and 15.39%, while those associated with regulatory spectra, relatively the same for the three categories of site, are between 2.39% and 13.39. Fluctuations of extrema around the mean spectrum values are quantified by the average value μ_{Crep} of the coefficient of variation C_{rep} . We can see that fluctuations associated to the real PSA ($\mu_{Crep} = 7.70\%$) are relatively less pronounced than those associated with regulatory response spectra ($\mu_{Crep} \approx 11.2\%$). We also note from Table II, that the variation ranges of amplitudes associated with the RPA2003 are characterized by constant values regardless to the type of soil considered.

TABLE II :Statistical Characteristics Of The Coefficient Of Variation Cov Of The Amplitudes Of Rpa Spectra

Soil type	Rock (S1)	Firm (S2)	Soft(S3)	Very soft (S4)
Mean value of COV : \square_{Crep}	11.2%	11.19%	11.18%	11.19%
Standard-deviation value of COV : \square_{Crep}	3.62%	3.62%	3.63%	3.62%
Range of values of COV $C_{rep}(f)$	2.41% - 13.38%	2.39%- 13.38%	2.39%-13.34%	2.41%-13.39%

3.3 Sensitivity of the response to the uncertainty on the statistical characteristics of the damping

In this section, we study the sensitivity of the seismic response to changes in the values of C_{ξ} . The test results on real buildings showed that C_{ξ} varies between [33% - 87%], which

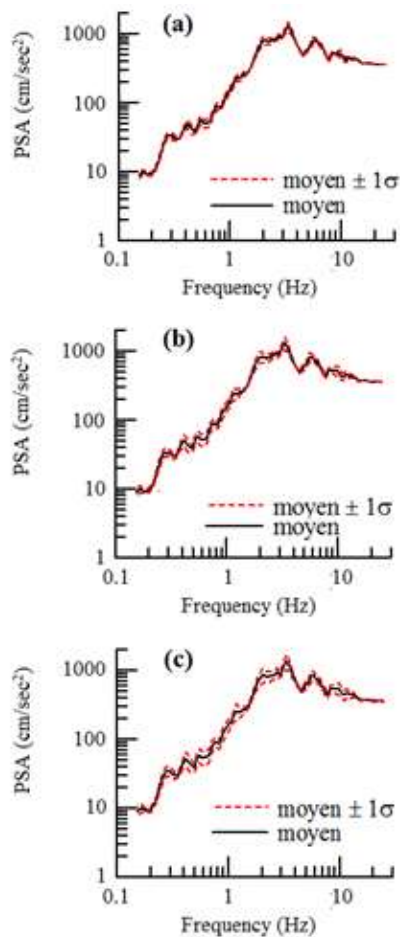


Fig. 5: RPA spectrum (soft ground) mean \pm 1 standard deviation for different values of C_{ξ} : a) $C_{\xi} = 40\%$, b) $C_{\xi} = 60\%$, c) $C_{\xi} = 80\%$.

TABLE IV: Values Of The Cov Of Amplitudes Of Rpa Spectrum (Firm Soil S3).

Values of C_{ξ} (COV of damping values)	40 %	60 %	80 %
Values of $\mu_{C_{rep}}$ (mean of COV of spectral amplitudes)	11.19%	15.41%	18.50%
Values of $\sigma_{C_{rep}}$ (standard-deviation of COV of spectral amplitudes)	3.63%	4.95 %	5.88%
Range of values of C_{rep} (COV of spectral amplitudes)	2.38%-13.29%	3.416%-18.29%	4.13%-21.94%

These same results are also put in evidence by Table IV which show values obtained for design spectrum associated to firm soil (S3). We can see that values of $\mu_{C_{rep}}$, the mean of the spectral amplitudes, grow from 11.19% to 18.50%, when the value of C_{ξ} increases from 40% to 80. The same observation can be made when we analyse the variations of the values of the standard - deviation $\sigma_{C_{rep}}$ and those of the extrema ranges of C_{rep} which respectively grow from 3.63% to 5.88% and from 40% to 80% for increasing values of C_{ξ} .

4. Conclusion

In this work we investigated the effect of uncertainties related to the damping of the structural seismic response. The damping of the structure, due in part to the material and links, is modeled by a random variable with a given variability and the seismic responses of structures, expressed in terms of real spectra, are estimated by using the Monte Carlo method. The applicability of the proposed methodology is illustrated using the target response spectrum corresponding to a record of the Parkfield Earthquake of June 27, 1966 and design response spectra of the Algerian Earthquake Regulations (RPA 2003)

The results show two frequency ranges [~ 3 Hz] and [$> \sim 8$ Hz] associated with rigid and flexible structures, respectively, where the-unnoticeable-influence of damping results in small fluctuations of the responses around their respective averages. The largest fluctuations are obtained for the intermediate frequency range [~ 3 Hz - 8Hz \sim] for which the damping effect is more significant.

It has also been shown that when the variability of the damping increases, characterized by increasing values of COV CS, the amplitudes of the spectral response also increase. This foreseen outcome reflects the fact that the amplitude levels of the response are inversely proportional to the damping values.

This study can be extended when taking into account the effects of systematic uncertainties induced by the engineer in the values used for the dynamic parameters (mass, stiffness, natural frequency of vibration). The study can also be extended to investigate the influence of variations of the damping on the dynamic response of the structure by considering the aspects of the different seismic regulations in force around the world.

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