Amplitude and Frequency Estimation of Power System Harmonics Using Adaptive Hopf Oscillator

Suleiman Sabo Kurawa
suleimankurawa@gmail.com

Abstract: Harmonics constitute a great problem to the power quality of power systems. In order to mitigate the effect of these harmonics, it is important to be able to estimate the parameters of the harmonics. This paper proposes the use of an adaptive hopf oscillator (AHO) to estimate the frequency and amplitude of power system harmonics. The method exploits the frequency and amplitude learning capability of the AHO. Since the power system signal is usually a multi-frequency component signal, a pool of AHOs in a negative feedback loop will be used so that each AHO will learn a particular frequency component. The AHOs in the pool are not coupled and each AHO learns a different harmonic independent of the other. The proposed method has the ability to estimate the parameters of both a pure power system signal and when the signal is corrupted with white noise. The method can also estimate the parameters of inter-harmonics and sub-harmonics. The effect of amplitude and frequency initialization of the AHO on the estimation will also be explored.

Keywords: harmonics, sub-harmonics, inter-harmonics, adaptive hopf oscillator, power systems

1. Introduction

Nonlinear loads and time varying devices such as arc furnace, rectifiers, uninterruptable power supply, thyristors, diodes e.t.c have become ubiquitous features of an electric power system. As a result of this, periodic distortion such as harmonics, inter– harmonics and sub - harmonics are induced into both the current and voltage waveform of the electric power system. These harmonics degrade the power quality of power systems. Harmonics can also cause problems such as loss of efficiency in generator, saturation and copper losses in transformers, relay misoperation and voltage dips. Therefore, to improve the quality of the power system signal, these harmonics need to be estimated so that an appropriate filter or compensator can be designed to curtail the effects of the harmonics.

Several algorithms have been proposed in estimating the harmonics of power systems. Earlier algorithms widely employed the use of Fast Fourier Transform (FFT) in estimating the harmonics. However, due to spectrum leakage effect of FFT, such algorithms did not give good parameter estimates. Kalman filter [1] [2] [3] [4] has also been used in estimating power system harmonics. However, algorithms employing the kalman filter may encounter some problems when tracking dynamic signals. Recently, advance techniques such as neural network [5] [6] [7] and genetic algorithm [8] [9] have been used in estimating the harmonics of the power signal. Although these contemporary techniques provided better parameter estimations, they usually have slow rate of convergence.

Other algorithms also used in estimating the harmonics include singular value decomposition and least square method [10], reduced order state space model and set theory [11], to name but a few.

AHO [12] [13] [14] is widely used in the field of robotics. In [15] [16], the AHO was used as a controller for locomotion control, while in [17] it was used as the building block for central pattern generators. AHO have also been used in estimating periodic disturbance to improve the efficiency of autonomous underwater vehicle controllers [18] [19]. The ability of the AHO to perform dynamic frequency analysis was outlined in [20]. AHO is also capable of trajectory generation [13].
2. Adaptive hopf oscillator (AHO)

AHO has a correlation – based type of learning similar to that observed in neural networks. The learning was therefore termed as dynamic hebbian learning [12]. Because of this correlation, the frequency of the AHO first adapt to the required value before the amplitude starts learning. The AHO varies from the traditional hopf oscillator because the intrinsic frequency of the oscillator is made to be a dynamic state of the system. This means that the intrinsic frequency of the AHO can adapt to the frequency of any teaching signal.

\[
\dot{r}(t) = \gamma (\mu - r^2(t)) r(t) + \epsilon F(t) \cos \varphi(t) \tag{1}
\]

\[
\phi(t) = \omega(t) - \frac{\epsilon}{r(t)} F(t) \sin \varphi(t) \tag{2}
\]

\[
\dot{\omega}(t) = -\epsilon F(t) \sin \varphi(t) \tag{3}
\]

\[
\dot{G}(t) = \rho F(t) \cos(\varphi(t)) r(t) \tag{4}
\]

\[
G = \alpha(t) r(t) \cos \varphi(t) \tag{5}
\]

\(r(t)\) is the radius of the limit cycle, \(\mu > 0\) controls the radius of the limit cycle, \(F(t)\) is the driving signal or perturbation, \(\varphi\) determines the how fast the oscillator returns to the limit cycle after perturbation and \(\varphi(t)\) is the phase of the oscillator. \(\omega(t)\) and \(\alpha(t)\) represent the frequency and amplitude of the oscillator respectively while \(G(t)\) is the output of the oscillator. The frequency and amplitude learning rates are represented by \(\epsilon\) and \(\rho\) respectively. These values determine how fast the AHO converges to frequency and amplitude of the teaching signal. The proof of convergences of the AHO can be found [12].

The AHO can be used as the building block of a system capable of performing dynamic frequency analysis [20]. The system is created by using a pool of the N AHO via a negative feedback. The perturbation to the pool of AHOs \(F(t)\) is the teaching signal minus the learned signal (which is the summation of the output of each AHO in the pool).

![Diagram of A pool of N AHOs with no coupling term. This is the same configuration as in [18] [19]. In such feedback structure, each AHO is independent of the other and can track its own frequency component.](http://dx.doi.org/10.17758/UR.U0915108)
The concept behind the pool of adaptive oscillator is that when an AHO learns a particular frequency component of $I_{\text{teach}}$, it becomes part of the feedback signal. It gets eliminated from the perturbation signal $F(t)$ via the negative feedback. The pool of AHOs will therefore be perturbed with only the part of the $I_{\text{teach}}$ that has not already been learned by one of the AHO in the pool. In this way, all the frequency component within $I_{\text{teach}}$ can be learned by the AHOs and the frequency spectrum of the signal can be reconstructed.

To learn a signal with completely unknown frequency and amplitude components, as many AHOs (with either uniform or random frequency initialization) has to be used in the pool until the error between $I_{\text{teach}}$ and $L_{\text{learned}}$ is small (ideally zero). This may not be ideal since the pool may require a very large number of AHOs due to many oscillators learning the same frequency component of $I_{\text{teach}}$. This will lead to an increase in computation time and hardware necessary to implement the pool of AHOs. Therefore, an optional but important step is to perform a Fast Fourier Transform of the teaching signal. This will help in initializing the AHOs with frequency values close to the frequency components in the teaching signal and:

- speed up the rate of convergence of the AHOs in the pool
- limit the number of AHOs in the pool to only the number of frequency components in the teaching signal, thus reducing the number of AHOs in the pool

3. Power System Signal Modeling

The voltage or current of a power system can be described as

$$y(k) = \sum_{i=1}^{n} A_i \sin(\omega_i t + \varphi_i) + k_g e(k)$$

(6)

where $A_i$, $\varphi_i$, and $\omega_i$ are the amplitude, phase and angular frequency of the $i^{th}$ harmonic, $e(k)$ is the Gaussian white noise with zero mean and unit frequency, while $k_g$ is the noise gain factor and $n$ is the number of harmonics in the signal. The angular frequency is given by

$$\omega_i = 2\pi f_0$$

(7)

Where $f_0$ is the fundamental frequency.

Three different signals will be considered in evaluating the efficiency of this estimation technique. The first signal is a static signal that contains only harmonics. The second signal is a time varying signal which also contains only harmonics. The third signal is a static signal which contains inter - harmonics and sub - harmonics. Since the estimation is concerned with frequency and amplitude, the power system signals will not be modeled with the phase.

4. Simulation

4.1 Static signal

The signal used for this simulation is

$$y(t) = 1.5 \sin(\omega t) + 2.8 \sin(3\omega t) + 0.6 \sin(5\omega t) + 0.35 \sin(7\omega t) + 0.07 \sin(11\omega t) + 0.05 \text{rand}(t)$$

The fundamental frequency is $f_0 = 50\text{Hz}$, while the $0.05 \text{rand}(t)$ is the white noise that corrupts the signal. The signal contains the fundamental, third, fifth, seventh and eleventh harmonics.
Fig 2: Frequency estimation of the harmonics in the static signal. The oscillator parameters are: $\varepsilon = 80$, $\rho = 1$, while $\mu = 1$ and $\gamma = 20$. The initial value of the intrinsic frequency of each AHO is: 

$$\omega_2(0) = 295, \omega_2(0) = 920, \omega_2(0) = 1550, \omega_2(0) = 2180, \omega_5(0) = 3450.$$ 

All the initial values of the intrinsic frequency of the AHOs are in $\text{rad/s}$. 

Fig 3: Amplitude estimation of the harmonics in the static signal. The oscillator parameters are: $\varepsilon = 80$, $\rho = 1$, while $\mu = 1$ and $\gamma = 20$. The initial value of the amplitude of each AHO is: 

$$\alpha_2(0) = 0, \alpha_2(0) = 0, \alpha_5(0) = 0, \alpha_5(0) = 0, \alpha_5(0) = 0.$$ 

Fig 2 shows that the AHO can accurately estimate the frequencies (50Hz, 150Hz, 250Hz, 350Hz, 550Hz) of all the harmonics in the power system signal. The intrinsic frequency of each AHO was initialized with a value close to one of the frequency of the harmonics in the power system signal. This ensures that each AHO will track a different frequency component and hence limit the number of AHO in the pool to be equal to the number of harmonics in the signal.

Fig 3 shows the estimation of the amplitudes of the different harmonics in the power system signal. Due to the correlation between the amplitude and the frequency of the oscillator, where the frequency of the oscillator learns first before the amplitude, all the AHOs in the pool can be initialized with the same value. Once an AHO learns the frequency of a particular harmonic, the amplitude of that AHO will automatically learn the amplitude of the harmonic whose frequency was learnt by the AHO. The amplitudes of the fundamental, third, fifth, seventh and eleventh harmonics are 1.5, 2.8, 0.54, 0.35 and 0.07 respectively. The amplitude of the fundamental harmonic has no offset in the estimation while the other amplitudes were estimated with some offset from the real value.
a. Power system signal with inter-harmonics and sub-harmonics

The signal to be considered is

\[ y(t) = 1.2 \cos(1.23\omega t) + 0.5 \cos(2.06\omega t) + 0.25 \cos(1.78\omega t) + 0.8\cos(0.5\omega t) \]

This signal contains three inter-harmonics \((1.23\omega t, 2.06\omega t, 1.78\omega t)\) and one sub-harmonics \((0.5\omega t)\), and the fundamental frequency is also \(f_0 = 50\text{Hz}\). Here, the signal is not corrupted with white noise.

![Graph](http://dx.doi.org/10.17758/UR.U0915108)

Fig 4: Frequency estimation of inter-harmonics and sub-harmonic of a power system signal. The oscillator parameters are: \(\epsilon = 80, \rho = 1\), while \(\mu = 1\) and \(\gamma = 20\). The initial value of the intrinsic frequency of each AHO is:

\[ \omega_1(0) = 380, \omega_2(0) = 6300, \omega_3(0) = 540, \omega_4(0) = 140. \]

All the initial values of the intrinsic frequency of the AHOs are in \(\text{rad/s}\).

![Graph](http://dx.doi.org/10.17758/UR.U0915108)

Fig 5: Amplitude estimation of inter-harmonics and sub-harmonic of a power system signal. The oscillator parameters are: \(\epsilon = 80, \rho = 1\), while \(\mu = 1\) and \(\gamma = 20\). The initial value of the amplitude of each AHO is:

\[ \alpha_1(0) = 0, \alpha_2(0) = 0, \alpha_3(0) = 0, \alpha_4(0) = 0. \]

Fig 4 and 5 showed that the AHO is capable of estimating the frequency and amplitude of the inter-harmonics and sub-harmonic of the power system signal. As with the static signal, the frequency estimation is more accurate than the amplitude estimation. The estimated frequencies are 62Hz, 103Hz, 89Hz and 25Hz. These are the exact frequencies of the inter-harmonics and sub-harmonic of the power system signal. The estimated amplitudes are 1.1924, 0.4913, 0.2467 and 0.8. The accuracy of the amplitude estimation is better than in the case of the static signal which was corrupted with white noise.

Also, since the initial frequencies of the AHOs were close to that of the inter-harmonics and sub-harmonic in the signal, only four AHOs were required in the pool. This showed that each AHO in the pool tracked a different frequency component.

b. Time-varying signal

The time-varying signal used in the simulation is of the form

\[ y(t) = (2.5 + x_1(t))\sin(\omega t) + (1.8 + x_2(t))\sin(3\omega t) + (0.5 + x_3(t))\sin(5\omega t) + 0.005\text{rand}(t) \]

where

\[ x_1 = 0.18\sin(2\pi f_1 t) + 0.08\sin(2\pi f_3 t) \]
\[ x_2 = 0.08\sin(2\pi f_2 t) + 0.04\sin(2\pi f_3 t) \]
\[ x_3 = 0.025\sin(2\pi f_1 t) + 0.006\sin(2\pi f_3 t) \]
The fundamental frequency and Gaussian noise is the same as that used for the static signal.

$$f_1 = 1Hz, f_2 = 3Hz, f_3 = 6Hz$$

The oscillator parameters are: $\varepsilon = 80, \rho = 0.95$, while $\mu = 1$ and $\gamma = 20$. The initial value of the intrinsic frequency of each AHO is: $\omega_1(0) = 295, \omega_2(0) = 920, \omega_3(0) = 1560$. All the initial values of the intrinsic frequency of the AHOs are in rad/s.

Fig 5: Frequency estimation of the harmonics in the static signal. The oscillator parameters are: $\varepsilon = 80, \rho = 0.95$, while $\mu = 1$ and $\gamma = 20$. The initial value of the intrinsic frequency of each AHO: $\omega_1(0) = 295, \omega_2(0) = 920, \omega_3(0) = 1560$. All the initial values of the intrinsic frequency of the AHOs are in rad/s.

Fig 6: Amplitude estimation of the harmonics in the static signal. The oscillator parameters are: $\varepsilon = 80, \rho = 0.95$, while $\mu = 1$ and $\gamma = 20$. The initial value of the amplitude of each AHO: $\alpha_1(0) = 0, \alpha_2(0) = 0, \alpha_3(0) = 0, \alpha_4(0) = 0, \alpha_5(0) = 0$.

Fig 5 showed that the AHO can estimated the frequencies of the harmonics of a time–varying signal with good accuracy even when the signal is corrupted with noise. For the amplitude estimation, the estimated amplitude of the fundamental harmonic oscillates around 2.5, which is the actual value, while the estimated amplitudes of the third and fifth harmonic oscillates around 1.732 and 0.45 respectively. As with the other signals, the amplitude estimation of the fundamental harmonic has a greater degree of accuracy than the other harmonics. Also, the accuracy of the amplitude estimation is less when compared with the power system signal that was not corrupted with Gaussian noise.

5. Conclusion

The estimation scheme using a pool of AHOs presented was shown to not only be capable of estimating the harmonics of a power system signal, but it could also estimate the inter–harmonics and sub–harmonics. The estimation scheme was also shown to be robust to different types of signals. Moreover, the estimation scheme had a good degree of accuracy even when the power system signal was corrupted with Gaussian noise. The effect of the frequency initialization on the number of AHOs to be used in the pool was also examined. Initializing the frequencies of the AHOs to values close to the frequencies of harmonics in the power system signal will limit the number of the AHOs to be equal to the number of harmonics present in the power system signal. The correlation between the frequency of the AHO and its amplitude means that the amplitude can be initialized with any value, and once the frequency of the AHO learns the frequency of a particular harmonic, the amplitude of the AHO will automatically learn the amplitude of that harmonic.

6. Bibliography


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