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Stochastic Modelisation of Dynamic Response of Structures with Uncertain Damping

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Abstract**:** *Damping plays a key role in controlling the overall design of dynamically sensitive structures. However, the inherent uncertainty in estimation and prediction of damping in structural systems poses a difficult problem in structural dynamics since structural damping has intrinsic variability and depends on a wide range of factors. This paper utilizes two probabilistic modeling techniques to describe the variability of structural damping and to investigate its impact on the sensitivity of dynamic response of MDOF systems with random damping. Numerical results show that damping variability has significant effect on dynamic response especially for lightly damped structures and high variability of damping values in the neighborhood of a resonant frequency. For small values of damping uncertainty, the linearization technique is found to be adequate whereas for larger values, a higher order statistical model is required and statistical distributions other than the normal distribution should be considered.*

Keywords: *Random damping, dynamic response, sensitivity, Monte Carlo simulation, structures*

1. Introduction

The problem of dynamic response of multi-degrees of freedom (MDOF) systems with random damping has received significant attention in recent years (e.g.[1], [2], [3], [4]). This problem is of importance in relation to the design of dynamically sensitive structures such as tall buildings and industrial chimney stalks that strongly rely on damping for their performance under wind and seismic induced vibrations.

However, selecting an appropriate damping value to be used for the design of a particular modern high rise building is to some extent a subject of controversy. It is interesting to note that the last 30 years of study of damping in full-scale structures no consensus view has been achieved. A controversy has occurred because of paucity of measured data from real buildings, large variance errors in many of the few measurements that were taken, and the occasional misuse of measurement techniques.

The inherent uncertainty in estimation and prediction of damping in structural systems, poses a difficult problem in structural dynamics since damping, unlike other system parameters such as mass and structural stiffness, does not refer to a single physical phenomenon. Damping estimates have intrinsic variability and may depend on a wide range of factors including vibration amplitude, natural frequency, structural dimensions, local site conditions [5].

The difficulty of an accurate prediction of damping emphasizes the uncertainty involved in representing the response of the system. For excitation frequencies close to the resonant frequencies, the sensitivity of the response to damping becomes critical. Errors in the estimation of the damping matrix will generally result in large error in the response.

In order to provide additional information for practical applications in engineering design, this article presents the main results of a numerical investigation on the sensitivity of dynamic response of MDOF systems with random damping. A statistical linearization technique coupled with the use of sensitivity functions has been utilized to determine second order statistics of the system response. The numerical results obtained from the application of this methodology to a typical industrial building structure have been checked by Monte Carlo simulation method. The significance of damping randomness and its implications on the sensitivity of system response in the neighborhood of a resonant frequency are discussed in light of considerable ranges of damping uncertainties. The limits of approximation of the statistical linearization technique are also established.

2. Background

2.1. Statistical linear model

The matrix system of differential equations of motion governing the displacement {x(t)} an n-Multi-Degree Of Freedom (n-MDOF) dynamic system subjected to an external excitation {F(t)} can be written as:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{F(t)\} \qquad (1)$$

A physical system described by (1) and representative of a typical industrial building with rigid floors is shown in Fig 1, for n=5. In this system [M] and [K] are deterministic: It can be shown that the damping matrix [C] depends linearly on the viscous damping coefficients ci according to the definition of its elements.

$$C_{ij} = \begin{cases} c_i + c_{i+1} & \text{if } i-j=0 \\ -c_i & \text{if } i-j=1 \\ -c_j & \text{if } j-i=1 \\ 0 & \text{if } |i-j|>0 \end{cases}$$

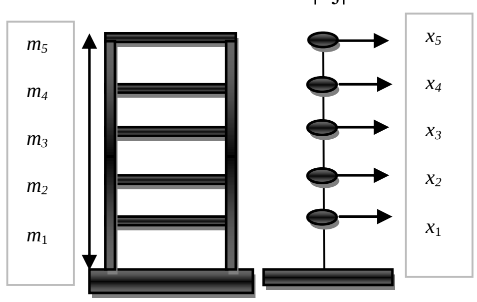


Fig. 1: Five story industrial building

The damping coefficients c_i are assumed to be normally distributed with mean value $\bar{c}_i$ and standard deviation σ_{ci}. The total number of dampers, in the general case, is $p = n$. For the sake of convenience, it will be convenient to arrange the damping coefficients in a p dimensional vector $\{c\}$. Denoting by $\{x^0\}$, the value of $\{x\}$ when $\{c\}$ takes on its mean value $\{\bar{c}\}$, and neglecting the higher order terms, the vector $\{x\}$ can be expanded in a Taylor series about $\{c\}=\{\bar{c}\}$ as follows:

$$\{\Delta x\}=\partial\{x\}/\partial c_1.\Delta\Delta_1+\partial\{x\}/\partial c_2.\Delta\Delta_2+.......+\partial\{x\}/\partial c_p.\Delta\Delta_p \qquad (2)$$

Introducing sensitivity functions

$$\xi_{ij}=(\partial x_i/\partial c_j)_{c_k=\bar{c}_k} \text{ (k=1 ...p)} \qquad (3)$$

Eq. (2) can be rewritten in matrix from:

$$\{\Delta x\}=[\xi]\{\Delta c\} \text{(i=1,...,n ; j=1,...,p)} \qquad (4)$$

Denoting with $\{\xi\}_i^T$ the i[th] row and with $\{\xi\}_j$ the j[th] column of the Jacobian matrix $[\xi]$ we can write the i[th] component of (4) as follows:

$$(x_i-x_i^0)=\{\xi\}_i^T\{c_j-\bar{c}_j\}=\{c_j-\bar{c}_j\}^T\{\xi\}_i \qquad (5)$$

From (5), we obtain

$$(x_i-x_i^0)^2=\{\xi\}_i^T\{c_j-\bar{c}_j\}\{c_j-\bar{c}_j\}^T\{\xi\}_i \qquad (6)$$

The product $\{c_j-\bar{c}_j\}\{c_j-\bar{c}_j\}^T$ generates a p by p symmetric matrix which is statistically variable while the terms containing the sensitivity functions are statistically constant since, according to (3) they are evaluated at $\{c\}=\{\bar{c}\}$. If we take the expected value of (6), we obtain

$$\sigma_{x_i}^2=\{\xi\}_i^T\,[\,\mathrm{cov}\,]\,\{\xi\}_i \tag{7}$$

where $\sigma_{x_i}^2$ denotes the variance of the random variable x_i, and *[COV]* is the covariance matrix of the damping coefficients c_i. Its ij[th] element is defined by

$$\mathrm{cov}_{ij}=\rho_{ij}\sigma_{ci}\sigma_{cj} \tag{8}$$

Where $\rho_{ij}=1$ if i=j

The elements of the main diagonal are the variance and are known from the assumed distribution law. The off diagonal elements contain the correlation ρ_{ij} between the various damping coefficients.

Various assumptions can be made regarding the correlation between damping coefficients. The simplest one is to treat the damping coefficients as uncorrelated random variables by setting the off diagonal elements of covariance matrix to zero.

The sensitivity functions ζ_{ij} are available by differentiating (1) with respect to c_j as follows:

$$\left.\begin{array}{c}[\mathrm{M}]\{\partial\ddot{\mathrm{x}}/\partial\mathrm{c_j}\}+[\mathrm{C}]\{\partial\dot{\mathrm{x}}/\partial\mathrm{c_j}\}+[\mathrm{K}]\{\partial\mathrm{x}/\partial\mathrm{c_j}\}=-(\partial[\mathrm{C}]/\partial\mathrm{c_j})\{\dot{\mathrm{x}}\}\\ \text{or}\\ [\mathrm{M}]\{\ddot{\xi}\}+[\mathrm{C}]\{\dot{\xi}\}+[\mathrm{K}]\{\xi\}=-(\partial[\mathrm{C}]/\partial\mathrm{c_j})\{\dot{\mathrm{x}}\}\end{array}\right\} \tag{9}$$

The left side of (9) is identical to (1) and the right side ($-(\partial[\mathbf{C}]/\partial c_j)\{\dot{x}\}$) can be interpreted as a fictitious a forcing vector. In the case of general loading {F}, the right side of (9) can be obtained from the time derivative $\{\dot{x}\}$ of the response $\{x_0\}$. For large MDOF systems, the vector $\{x_0\}$ can be computed by solving (1) with nominal damping $c_j=\bar{c}_j$ using mode superposition analysis [6]. Alternatively, the vectors $\{x\}$ and $\{\dot{x}\}$ can be obtained systematically by using step by step integration methods of structural dynamics. In order to obtain the vectors $\{\xi\}_j$ (j=1 …p), (1) must be solved first with the forcing vector $\{F\}$, after which (9) is solved p times.

Thus, the global bulk of computations involves essentially, a computer simulation for the evaluation of the covariance matrix **[cov]**, the solution of (1) for $\{x_0\}$, $\{\dot{x}_0\}$ and finally the solution for $\{\xi\}_j$, p times.

2.2. Monte Carlo simulation

In order to assess the limits of approximation of the statistical linear model, the Monte Carlo simulation technique is used. This technique consists essentially in generating numerically statistical results of the response without performing any physical experimentation. The computer simulation involves sampling at random to simulate numerically large number of experiments and to observe the results. In the present case, a sequence of damping coefficients normally distributed with prescribed mean and standard deviation is first generated. The dynamic system is then solved for each value of the random variable ci to give a sample value of response xi .These sample values are finally used to determinate the second order statistics of the response. The method constitutes a good test for the theory.

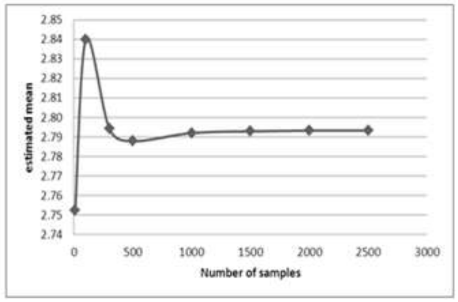


Fig. 2: Convergence of mean response estimate with sample size

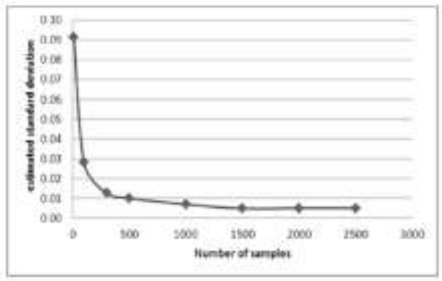


Fig. 3: The computational process, however, is very long and expensive: Eq.1 must be solved a number of times equal to the number of iterations.

In the example presented here, this number was set to one thousand five hundred in accordance with the progressive results obtained for the mean response estimate, its standard deviation and hence the corresponding confidence interval as a function of the number of samples. Typical convergence of these estimates with increasing sample size is illustrated in Fig. 2 and Fig. 3 respectively. In other cases, however, the slow convergence of statistical processes may require even more iterations. The savings in computer time achieved with the linear model become quite evident.

For all purposes deemed useful, the probabilistic description in terms of second order statistics is illustrated in Fig 4 for the histograms and approximate PDFs of the damping coefficients and in Fig. 5 for the maximum dynamic response, respectively.

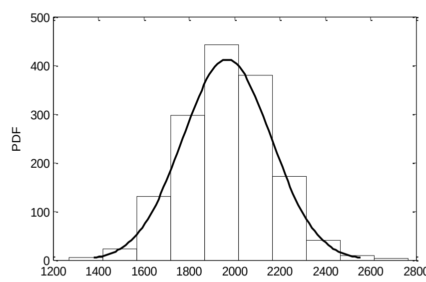
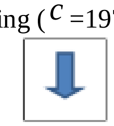


Fig. 4: Histogram and PDF of damping ($\bar{c}$ =1971, 86KN/m/s, COV=0.1; see Table I)



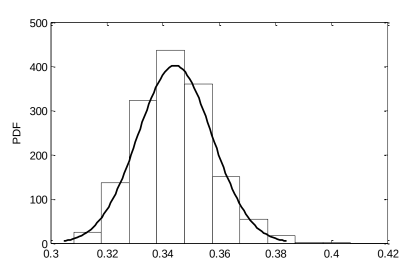


Fig. 5: Histogram and PDF of response atfirst floor (=0.34cm, COV=0.04;see TableI)

3. Numerical Results And Discussion

A numerical example is now presented to illustrate the methodology described herein and to demonstrate quantitatively the influence of uncertainty level in damping on overall response of a structural system subjected to dynamic excitation. The structural system shown in Fig. 1 is taken as basic model for the computations. For the sake of clarity, the lumped mass at each floor, the inter-story stiffness and the mean values of inter-story damping constants between each level are kept in this study, constant, although different values could be used as input. The lumped mass at each floor is equal to $m_1 = m_2 = m_3 = m_4 = m_5 = 150$t and the inter-story stiffness between each level is such that $k_1 = k_2 = k_3 = k_4 = k_5 = 210*10^3$ KN/m. The mean values $\bar{c}$ of the inter-story damping constants, examined in this study are (in ascending order) $\bar{c}$ =197.19, $\bar{c}$ =394.37, $\bar{c}$ =1183.11, $\bar{c}$ =1577.48and $\bar{c}$ =1971.86KN/m/s respectively. Each value of the damping constant was assigned a COV that varied from 10% to 50% with uniform increments of 10%. A rotating machine, located at the centroid of the first story exerts a harmonic loading of amplitude F=560KN, and frequency Ω=31.09 rad/s corresponding to the second resonant frequency of the structural system.

In Table I, the mean value and the standard deviation of maximum displacement at each building lumped floor for several combinations of damping value c and COV are presented. The results obtained from the application of SLM are systematically compared with those derived from the MCS method.

It is noted that the two methods are in good agreement up to a COV of damping values less than 30%.

If the uncertainty about damping is such that larger COV values should be considered, the SLM becomes inadequate and higher order statistics should be considered. Moreover, the assumption of Gaussian distribution should be abandoned in order to prevent negative values of c, i.e. to ensure that damping must be always positive. Unfortunately, this condition cannot be satisfied "exactly" if the Gaussian distribution is used, unless, we choose *standard deviations less than one third of the mean*, in which case the probability of choosing a negative c is less than 0.0027.

On the other hand, it is seen that, if the COV of c is large (e.g: 50%), the errors between the two methods are considerably higher than in the other cases and the probability of choosing for c a negative value cannot be neglected. If this happens even few times during the MCS, a large dispersion in the computed response is very likely to occur. This situation is typical in presence of light damping and large COV (In the example considered, $\bar{c}$ =197.19 corresponds to five thousands of critical damping).

TABLE I: Comparison between Statistical Linear Model (SLM) and Monte Carlo Simulation (MCS)

Floor N°	Mean values of response		Error %	Standard deviations of response		Error %	Damping coefficients		COV	
	SLM	*MCS*		*SLM*	*MCS*		$\bar{c}$	σ_c	$\bar{x}/\sigma_x$	$\bar{c}/\sigma_c$
1	2.76	2.98	7.29	0.716	1.088	34.19			0.26	
2	3.6	3.88	7.32	0.937	1.427	34.33			0.26	
3	1.97	2.12	7.32	0.513	0.779	34.18	197.19	98.59	0.26	0.5
4	1.03	1.11	7.3	0.268	0.406	33.91			0.26	
5	3.31	3.57	7.32	0.861	1.312	34.35			0.26	
1	1.39	1.48	5.89	0.286	0.373	23.31			0.21	
2	1.8	1.91	5.97	0.374	0.493	24.05			0.21	
3	0.98	1.04	5.97	0.206	0.269	23.32	394.37	157.75	0.21	0.4
4	0.52	0.55	5.89	0.108	0.139	22.06			0.21	
5	1.65	1.75	5.99	0.344	0.454	24.17			0.21	
1	0.51	0.52	2.2	0.072	0.08	10.11			0.14	
2	0.6	0.61	2.48	0.093	0.104	10.37			0.16	
3	0.32	0.33	2.49	0.056	0.062	10.17	1183.11	354.93	0.17	0.3
4	0.19	0.19	2.21	0.03	0.033	9.71			0.16	
5	0.54	0.55	2.56	0.087	0.097	10.42			0.16	
1	0.4	0.41	1.23	0.037	0.039	6.28			0.09	
2	0.45	0.46	1.55	0.047	0.05	6.53			0.1	
3	0.24	0.25	1.58	0.03	0.032	6.32	1577.48	315.5	0.12	0.2
4	0.15	0.15	1.24	0.016	0.017	5.28			0.11	
5	0.39	0.4	1.65	0.044	0.047	6.95			0.11	
1	0.34	0.34	0.37	0.015	0.015	0.89			0.04	
2	0.36	0.36	0.51	0.019	0.019	1.21			0.05	
3	0.19	0.19	0.5	0.013	0.013	0.99	1971.86	197.19	0.07	0.1
4	0.13	0.13	0.38	0.007	0.007	0.46			0.05	
5	0.31	0.31	0.57	0.018	0.018	1.28			0.06	

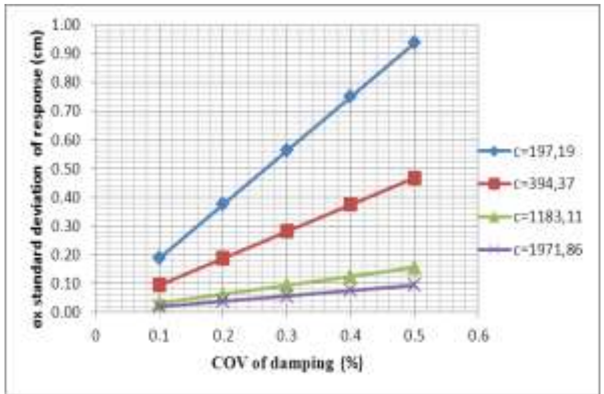


Fig. 6: Standard deviation of response (SLM) at2nd floor for different damping constants

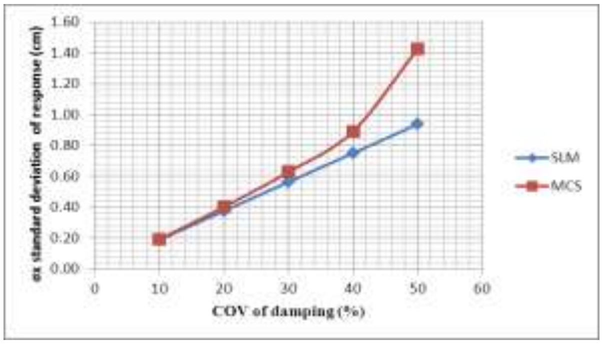


Fig. 7: Standard deviations of response (SLMand MCS) at 2nd floor (light damping, $\bar{c}$ =197.19)

In Fig. 6, the standard deviation of building response at the second floor (corresponding herein to the aximum floor displacement) as function of COV of damping is presented for different values of the mean. It is seen that the uncertainty in damping influences the system response. Depending on the mean value of damping the effects are more pronounced for higher variability of damping.

In Fig. 7, the standard deviation of building response with light damping at the second floor is presented for both SLM and MCS. The results suggests that differences in standard deviation of building response obtained for both methods are insignificant for small values of COV of damping. However, for larger values, the errors introduced by the linearization technique, increase concomitantly with an increase in COV of damping. Moreover, it should be noted that a *large dispersion in results between the SLM and MCS methods is observed for dynamic systems with light damping and large values of damping variability*. In such cases, a distribution different form the normal is likely to give more realist results.

4. Summary And Conclusions

This paper investigated the significance of damping variability on the dynamic response of MDOF building structures with uncertain damping in neighborhood of a resonant frequency.

A statistical linearization technique based on a Gaussian probability distribution of damping values coupled with the use of sensitivity functions has been utilized to determine second order statistics of the system response. The numerical results obtained from the application of this methodology to a typical industrial building structure have been checked by Monte Carlo simulation method and excellent agreement has been obtained even for considerable ranges of damping uncertainties. The limits of validity of the statistical linearization technique have also been determined. In addition, the significance of damping uncertainty on building response in the neighborhood of a resonant frequency has been illustrated for commonly associated damping uncertainties with structural damping values.

It is shown, among others, that depending on the mean values of damping, the effects are more pronounced for higher variability of damping values. The methodology used in this study is applicable, not only to damping but also to other structural parameters. The authors are currently expanding the present methodology to include the contribution of higher order terms in the statistical response model and to assess the influence of statistical distribution of uncertainty in damping (and other system parameters as well) using probabilistic distributions other than the normal distribution.

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