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Comparison of the Design of Reinforced Concrete Elements under Torsion According to Albanian Normative

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***Abstract*:** *In this work, the authors, presents the findings obtained from the analysis of calculating the torsion strength of reinforced concrete elements. The chose element, a reinforced concrete beam, is designed according to Albanian Normative with two methods, allowable stress design method and limit states design method. Furthermore, is made a comparison between the analysis results. In the beginning a presentation is made with the theoretical solution of the problem and after that the comparison is based on numerical solution. The conclusions are followed from the recommendations given in the end of this work.*

Keywords: *allowable stress design method, limit state design method, design of elements under torsion, Albanian Normative*

1. Introduction

The design of reinforced concrete structures, in different time periods, is performed in accordance with approved technical normative, using three methods [1]:

- allowable stress design method or classic method
- rupture method
- limit state design method

Together with the design against bending moment and shear force, a reinforced concrete element must be designed also from torsion, because this element could fail from the main tensile stresses, caused from torsion. The comparison of the results of the calculation against torsion of the same element according to the two methods, and the respective results are the aims of this study.

2. Design Methods of Reinforced Concrete Elements According to Albanian Normative.

2.1. Symbols According the Two Methods

The symbols between {…} belong to the allowable stress design method. Only the symbols that differ are shown. The same symbols are not shown.

a_{sw} – reinforcing steel area of one of the legs of the stirrup ; $\{f_{a,st}\}$

n – number of stirrup legs, n = 2,3,4…

$A_{sw} = n \cdot a_{sw}$ – reinforcing steel area for one stirrup; $\{F_{a,st} = n \cdot f_{a,st}\}$

s_w – distance between stirrups $\{a_x\}$

M_{pd} – Acting torsion moment; {T}

$[M_{pd}]$ – torsion strength $\{T_u\}$

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2.2. Allowable Stress Design Method

Torsion is always accompanied with bending and shear. Calculation inside the boundary of linear relationship between stress and strains is based in the summarising of the effects. Torsion effect are valued apart and after that are summarising with the effects of shear or bending. Calculating beyond the boundary of elasticity (post elastic) must take in consideration the simultaneity of different effects, but the incompletes of theoretical and experimental models force the use of independent actions principle. An element with a rectangular cross section with dimensions b x h is taken in consideration. From tests is seen that from torsion spiral cracks along the circumference of the elements are formed. The cracks forms a 45^0 angle with the longitudinal axis and are caused from the main tensile stresses.

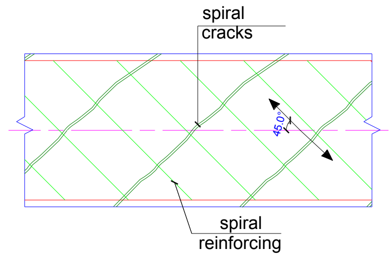

Fig. 2.2.1: Cracks and spiral reinforcing

The mos efficent reinforcement is showed in figure 2.2.1. Usually are placed stirrups and additional longitudinal reinforcing. See figure 2.2.2.

From the Construction Science is known that for a homogeny element, with rectangular cross section, the main tensile stresses from torsion are calculated with the following equation [1]:

$$\sigma_{kr,t} = \tau = M_{pd} / (C_1 \cdot b^2 \cdot h) \quad (2.2.1)$$

τ – maximum tangential stresses

C_1 – coefficent depending from the ration h/b, usually C_1=0.25

Tests had shown [1] that the measured values of main stresses are in average 1.6 times smaller than the calculated ones. Then:

$$\sigma_{kr,t} = \tau = M_{pd} / (1.6 \cdot 0.25 \cdot b^2 \cdot h) = M_{pd} / (0.4 \cdot b^2 \cdot h) \quad (2.2.2)$$

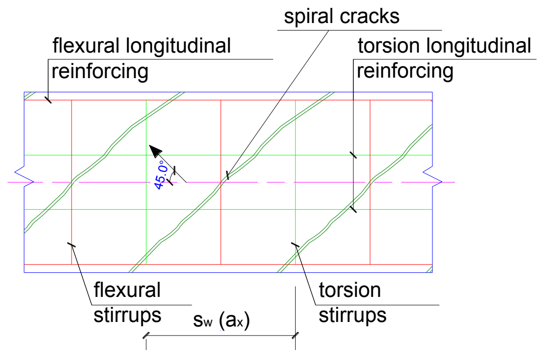

Fig. 2.2.1: Torsion transveres and longitudinal reinforcing

Sum of main stresses for torsion and shear must fulfill the condition (2.2.3) [1]:

$$\Sigma\sigma_{kr,t} \le \sigma_{bt}$$

If condition (2.2.3) is not fulfilled, than we should increase the dimensions of the cross section or the concrete mark.

If condition (2.2.4) is fulfilled, than there is no need to check the element from torsion.

$$\Sigma\sigma_{kr,t} \le \sigma_{bt,1} \quad (2.2.4)$$

If:

$$\sigma_{bt,1} < \Sigma\sigma_{kr,t} \le \sigma_{bt} \quad (2.2.5)$$

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Than the element must be designed from torsion.

Values of σ_{bt}, $\sigma_{bt,1}$ are given in the respective normative depending from the concrete mark.

To determine the area of one of the stirrup leg, in one of the element side, for a unit length:

$$fa,st = (Mpd \cdot ax) / (2 \cdot Fbr \cdot [\sigma a]) \quad (2.2.6)$$

Fbr – nucleus area

ax – distance between stirrups

$$Fbr = bbr \cdot hbr \quad (2.2.7)$$

Area of longitudinal reinforcing placed along the circumference of the cross section:

$$Fa,gj = (Mpd \cdot Ubr) / (2 \cdot Fbr \cdot [\sigma a]) \quad (2.2.8)$$

Ubr – nucleus circumference

$$Ubr = 2 \cdot (bbr + hbr) \quad (2.2.9)$$

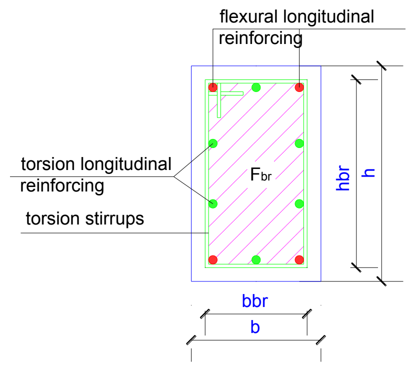


Fig. 2.2.3: Torsion transverse and longitudinal reinforcing, cross section nucleus

2.3. Limit State Design Method

According to Albanian normative [7], [8], [9] is studied the method of space cracks. Is accepted thet the stresses in longitudinal and transverse reinforcing, the are intercepted from the space cracks reaches respectively the values R_s and R_{sw}. Stresses in compressed concrete zone reach R_b.

R_b – concrete compressin strength

R_s – reinforcement tensile strength

R_{sw} – reinforcement strength for shear design, $R_{sw} = 0.8 \cdot R_s$

Failure is accepted like in the following figure 2.3.1.

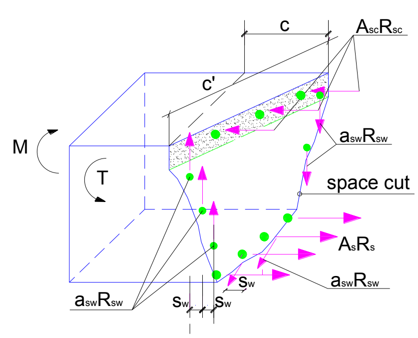


Fig. 2.3.1: Failure according to space cracks

Rsc – reinforcement compression strength

From the scheme of figure 2.3.1, three different modes derives. Scheme on figure 2.3.2a shows the case of simultaneous action of bending and torsion with zero shear force. Scheme on figure 2.3.2b shows the case of simultaneous action of torsion and shear force, but with benging in verys small values. Failure according to figure 2.3.2c shows the case when the element is under the action of bending (for a very small values of bending), but when in compression zone because of flexion the compression reinforcing is very small compared with the tensile reinforcing.

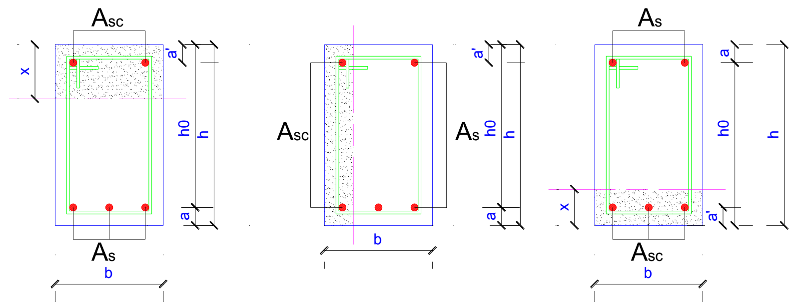


Fig. 2.3.2: Differend modes of failure

According to normative [8], [9] the design must performed for the three modes, depending from the position of the compressed zone of the space cut, accepting the smaller value of torsion strength. For the three modes the acting torsion moment must be smaller then the torsion strength:

$$T \leq \frac{R_s \cdot A_s \cdot (h_0 - 0.5x) \cdot \left(1 + \varphi_w \cdot \delta \cdot \lambda^2\right)}{\left(\varphi_q \cdot \lambda + \chi\right)} \qquad (2.3.1)$$

$$\lambda = \frac{c}{b} \quad (2.3.2) \qquad \delta = \frac{b}{2 \cdot h + b} \quad (2.3.3)$$

$$\varphi_w = \frac{b}{s_w} \cdot \frac{R_{sw} \cdot A_{sw}}{R_s \cdot A_s} \quad (2.3.4) \qquad \chi = \frac{M}{T} \quad (2.3.5)$$

$$\varphi_q = 1 + 0.5 \cdot h \cdot \frac{Q}{T} \quad (2.3.6)$$

M – bending moment

Q – shear force

A_s – tensile zone reinforcing area for every mode studied

A_{sc} - compressed zone reinforcing area for every mode studied

b, h – cross section dimension for every scheme studied.

c – projected length in longitudinal axis of the compressed zone.

x – compression zone depth calculated from equation:

$$A_s \cdot R_s - A_{sc} \cdot R_{sc} - b \cdot x \cdot R_b = 0 \quad (2.3.7)$$

For M = 0 and Q = 0 (pure torsion) we will have: $\chi = 0$ and $\varphi_q = 1.0$

According to scheme on figure 2.3.2a we will have:

$$\chi = \frac{M}{T} \quad (2.3.8) \qquad \varphi_q = 1.0 \quad (2.3.9)$$

For scheme 2.3.2b:

$$\chi = 0 \quad (2.3.10) \qquad \varphi_q = 1 + 0.5 \cdot h \cdot \frac{Q}{T} \qquad (2.3.11)$$

For scheme 2.3.2c:

$$\chi = -\frac{M}{T} \quad (2.3.12) \qquad \varphi_q=1.0 \quad (2.3.13)$$

The most dangerous cross section, with the smaller strength is the cross section with smaller projection “c”. It could be determined giving “c” different values (with step by step trial) in the calculation equation. Should:

$$0 \le c \le 2 \cdot h + b \quad (2.3.14)$$

Condition must be fulfilled:

$$\varphi_{w,min} \le \varphi_w \le \varphi_{w,max} \quad (2.3.15)$$

$$\varphi_{w,min} = 0.5 \cdot \left(1 - \frac{M}{M_u}\right) \quad (2.3.16)$$

$$\varphi_{w,max} = 1.5 \cdot (1 - \frac{M}{M_u}) \quad (2.3.17)$$

M_u – bending strength of the element

If $\varphi_w \le \varphi_{w,min}$, then in equations 2.3.1 and 2.3.7 $A_s \cdot R_s$ must be replicated with the decreasing ration $\frac{\varphi_w}{\varphi_{w,min}}$

If $T \le 0.1 \cdot b \cdot h_0^2 \cdot R_0$, then calculation is done according to the second scheme with the condition:

$$Q \le Q_w + Q_b - 3 \cdot \frac{T}{b} \quad (2.3.18)$$

Q_w – stirrups shear strength

Q_b – concrete shear strength

Concrete bearing capacity is against torsion “T” and bending “M” is ensured if the following condition is fulfilled:

$$T \le 0.1 \cdot b \cdot h_0^2 \cdot R_b \qquad (2.3.19)$$

Where $h \ge b$

2.4. Results of the Calculations of Flexural Reinforced Concrete Elements.

Numerical example 1

A beam under flexural action is analyzed. Rectangular cross section with b = 30cm and h = 50cm, a = a’ = 4cm, h_0 = h–a = 50–5 = 45cm. According to allowable stress design method concrete is of a grade M 300 (cubic resistance), σ_{bt} = 30daN/cm^2, σ_{bt1} = 12daN/cm^2; steel Ç.5, $[\sigma_a]$ = 1600daN/cm^2; According to limit state design method the concrete is of class B30 (cubic resistance), R_b = 160daN/cm^2, R_{bt} = 12.2daN/cm^2; steel Ç.5, R_s = 2400daN/cm^2, R_{sw} = 1920daN/cm^2; stirrups Φ8, s_w (a_x) = 10 cm, n = 2. Stirrups for bending Φ8, s_w (a_x) = 20 cm, n = 2.

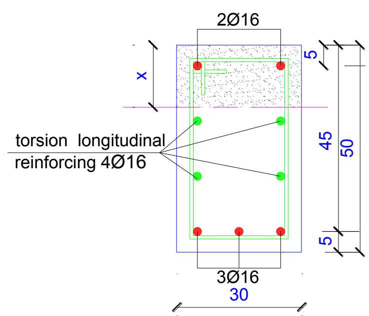


Fig. 2.4.1: Beam reinforcing

a) Allowable stress design method.

With equation (2.2.6) determine the torsion strength of stirrups

$[M_{pd}] = (2 \cdot F_{br} \cdot [\sigma_a] \cdot f_{a,st}) / a_x = 1280$daN·m.With help of equation (2.3.10) determine $c_0 = 133.6$cm.

With equation (2.2.8) determine the torsion strength of longitudinal reinforcing

$[M_{pd}] = (2 \cdot F_{br} \cdot [\sigma_a] \cdot F_{a,gj}) / U_{br} = 1715$daN·m.

The torsion strength of the element is the smaller of the two above results, $[M_{pd}] = 1280$daN·m.

b) Limit states design method.

Scheme a):

$Q = 0; M \neq 0; T \neq 0.$

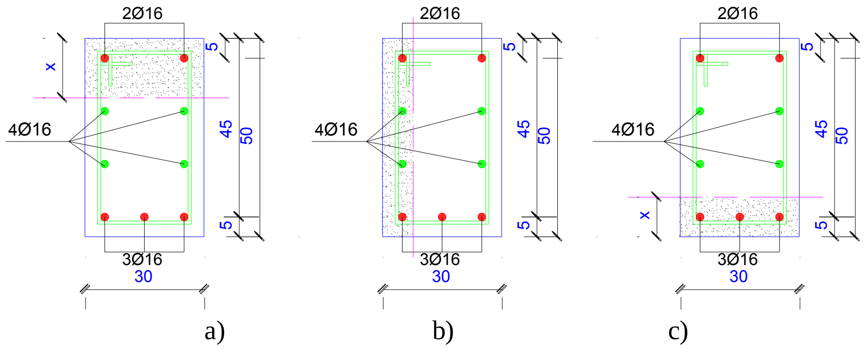
Let's accept $M = M_u = 6005$daN·m

With equation (2.3.7) determine x=1.0 cm

With equation (2.3.4) determine $\varphi_w=0.40$

With equation (2.3.16) determine $\varphi_{w,min}=0$

With equation (2.3.17) determine $\varphi_{w,max}=0$



Condition (2.3.15) is fulfilled $\varphi_w=0.40 > \varphi_{w,min}=0$

With equation (2.3.3) determine $\delta = 0.23$

With equation (2.3.14) determine c=130 cm

With equation (2.3.2) determine $\lambda = 4.33$

Let's accept $\chi = 2.2$, see equation (2.3.8)

With equation (2.3.6) determine $\varphi_q=1$

Right side of equation (2.3.1) represent torsion strength of the element T_u=2694 daNm

Scheme b):

$Q \neq 0; M = 0; T \neq 0.$

Let's accept $Q = Q_u = 6005$daN·m

Q_u – shear strength of the element

With the same calculation we find $T_u = 1071$daN·m.

Scheme c):

$Q = 0; M \neq 0; T \neq 0.$

With the same calculation we find $T_u = 2392$ daN·m.

The torsion strength of the element is the smaller of the above three results, $T_u = 1071$daN·m.

2.5. Calculations Analysis Results

To make possible the comparison of the calculation results, the elements are considered in the same conditions. The same class of concrete and steel is accepted, the same quantity of inclined reinforcement, the same distance between stirrups, the same stirrups diameter, the same cross section dimensions. Graphically is showed the connection between the torsion strength and factors as: cross section width, cross section height, allowable stresses of reinforcing steel, stirrups area, longitudinal reinforcing, shear force / torsion moment ratio. .

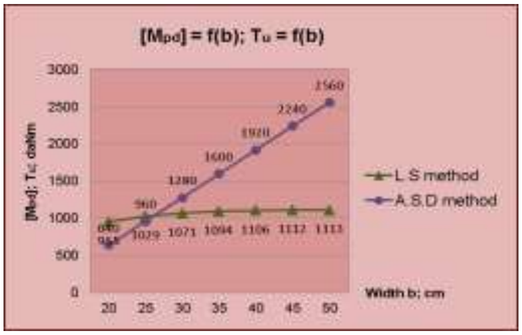


Fig. 2.5.1: Torsion strength and width relationship

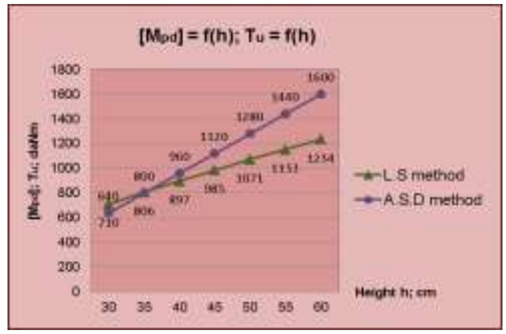


Fig. 2.5.2: Torsion strength and height relationship

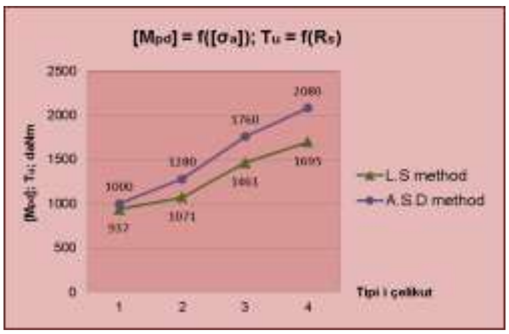


Fig. 2.5.3: Torsion strength and reinforcing strength relationship

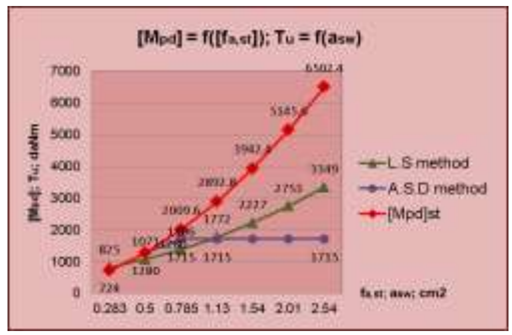


Fig. 2.5.4 Torsion strength and stirrups area relationship

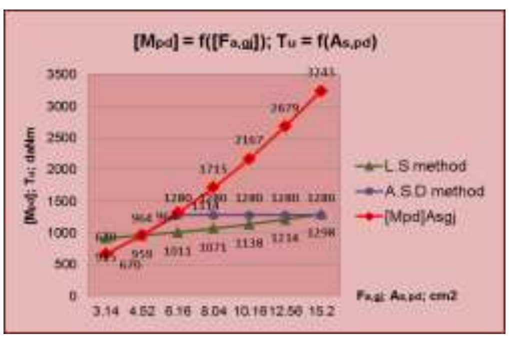


Fig. 2.5.5: Torsion strength and longitudinal reinforcing relationship

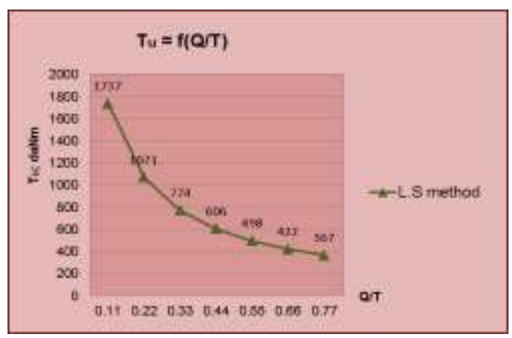


Fig. 2.5.5: Torsion strength and Q/T ratio relationship

3. Conclusions

- As is seen, the calculating formulas according to allowable stress design method are totally different from them of the limit state design method. Consequently the respective graphic are a lot different from each other. See figure 2.5.1 to 2.5.6.
- According to allowable stress design method, increasing of cross section width has a great effect in the increasing of the torsion strength of the element. When width is increased 2.5 time, torsion strength is increased 2.68 time. See figure 2.5.1.
- Increasing of cross section height brings the increasing of the torsion strength of the element, calculated with both the methods. According to allowable stress design method a 2 time increasing of cross section

height brings a 2.25 time increasing of torsion strength. According to limit state design method, a 2 time increasing of cross section height brings a 1.92 time increasing of torsion strength. Height increase is more effective to the allowable stress design method. The diagram is more inclined. See figure 2.5.2.

- Passing from type 1 to type 4 steel, its strength is increased. This brings the increasing of torsion strength. This increasing affect both the methods in the same way. Diagrams are almost parallel. See figure 2.5.3.
- Increasing of stirrups area (increasing of diameter) brings the increasing of torsion strength. See figure 2.5.4.
- According to allowable stress design method a 8.97 time increasing of stirrup area brings a 2.36 time increasing of torsion strength. Is seen that increasing of stirrup are from 0.785 cm^2 to 2.54 cm^2 do not affect the torsion strength and this is because of its conditioning from longitudinal reinforcing. See equations (2.2.6) and (2.2.8). In the meantime, increasing of stirrup area brings the increasing of respective torsion strength. See equation (2.2.6) and the red colored diagram in figure 2.5.4. A 8.97 time increasing of stirrups are brings a 8.97 time increasing of torsion strength.
- According to allowable stress design method a 8.97 increasing of stirrups area brings a 4.06 time increasing of torsion strength.
- Increasing of longitudinal reinforcing area brings a increasing of torsion strength. See figure 2.5.5.
- According to limit state design method a 4.84 time increasing of longitudinal reinforcing area brings a 1.91 time increasing of torsion strength. Is seen that with the increasing of longitudinal reinforcing area from 6.16 cm^2 to 15.2 cm^2, the torsion strength do not change because of the conditioning from the torsion strength of stirrups. See equation (2.2.6) and (2.2.8). In the meantime, increasing of longitudinal reinforcing brings the increasing of respective torsion strength. See equation (2.2.8) and red colored graphic in figure 2.5.5. A 4.84 time increasing of longitudinal reinforcing area brings a 4.84 time increasing of torsion strength.
- According to limit state design method a 4.84 time increasing of longitudinal reinforcing area brings a 1.41 time increasing of torsion strength.
- Figure 2.5.6 shows that, according to limit state design method, torsion strength depend also from the value of acting shear force. Increasing of shear force brings the decreasing of torsion strength. A 7 time increasing of shear force brings a 4.73 time the decreasing of torsion strength.
- According to allowable stress design method the presence of shear force do not affect the value of torsion strength.
- Different from the calculation for bending and shear, torsion strength, calculated with the limit state design method (L.S) is smaller than the torsion strength calculated with the allowable stress design method (A.S.D) See all the figures. This results was a little unexpected , knowing that L.S method use better the work of concrete and reinforcement in post elastic phase, We think that this result is affected from the complete different model and formulas used from the methods that we are comparing.
- As a conclusion, we can say, that the transition from the allowable stress design method to the limit state method, for the torsion design, according to Albanian Normative, brings the increasing of design safety, but simultaneous the cost increasing.

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