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Cracking in a Brittle Material

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Abstract**:** *The present study evaluates the stress field and energy during the cracking caused by applying pressure loading to a brittle material such as a glass. In this research study, the problem can be treated by Finite Element Method (FEM) using the Software (ABAQUS). Experimentally, semi-infinite thin rectangular elements containing an edge crack and having various cracks lengths are considered. For each crack's length, stress fields as well as the energy release rates are determined. Obtained results are compared and agreed with those of other researchers*

Keywords: *Cracks, Damage, Stress Intensity Factor, Amplification, Shielding, Energy Release Rate.*

1. Introduction

Microscopic observations of cracks propagation in brittle materials [1, 2] show that a damage zone develops in the vicinity of crack tips. This zone can be identified in certain cases like an intensive area of microcracks. These microcracks can have a significant influence on the propagation of the main crack.They can either cause crack amplification or crack shielding. Crack shielding reduces stress intensity factors of the main crack while crack amplification increases those values. These effects have been investigated by several researchers using exact analytical methods for some particular cases [3], and with analytical approximations under certain assumptions [4]. Because this damage can constitute an important toughening mechanism, problems dealing with crack propagation have received considerable research attention since they were introduced to fracture mechanics. As a result, a wide body of literature, on this topic, exists [5]. Solutions obtained are mostly based on the complex variable technique [6], or on numerical procedures [7], or asymptotic estimates for remotely located cracks [8]. Those techniques are usually different to a degree, but the basic principles remain the same.

The rate of the energy of restitution is defined as being the energy released during the propagation of the crack. For that, several researchers used the principle of the integral of Rice (J) in order to study the evolution of the crack. According to Griffith [9], the fracture is a phenomenon consuming energy and the difference is between the energy state of the atoms before and after cracking.

2. Stress Field

During the formation of a crack, a zone surrounding the initial crack is formed and where, a strong concentration of stress takes place. This zone is considered as being a zone with strong disturbance and commonly called a damage zone or fracture process zone. A great number of researchers admit nowadays that the extension of a crack is considered in a small zone close to the face of the crack in which, it exists high stress and separations where the mechanics of continuous mediums does not admit. On the other hand, around this zone, the remainder of the body whose behaviour is elastic or plastic concerns the mechanics of the continuous mediums.

2.1. Problem formulation

The problem is formulated in terms of complex potentials [10] using complex variable functions in plane elasticity, The stresses (σ_x, σ_y, σ_{xy}) are expressed in terms of the complex potentials $\phi(z)$ and $\omega(z)$ such that;

$$\sigma_x - \sigma_y + 2i\sigma_{xy} = 2\left(\overline{\phi(z)} - (z - \bar{z})\overline{\dot{\phi}(z)} - \omega'(\bar{z})\right) \quad (1)$$

$$\sigma_x + i\sigma_{xy} = \phi(z) + 2\overline{\phi(z)} - (z - \bar{z})\dot{\phi}(z) - \omega'(\bar{z}) \quad (2)$$

where; $\phi(z)$, $\omega(z)$ are complex potentials used in the representation of an elastic field z and σ_x, σ_y, σ_{xy} are the stress components. Then, the stress distribution near a crack tip under the traction free crack face can be expressed as [11]

$$\begin{bmatrix} \sigma_x & \sigma_{xy} \\ \sigma_{xy} & \sigma_y \end{bmatrix} = \frac{K_I}{\sqrt{2\pi r}} \begin{bmatrix} f_{11}(\theta) & f_{12}(\theta) \\ f_{21}(\theta) & f_{22}(\theta) \end{bmatrix} \quad (3)$$

Here, the first two terms in the expansion form are singular at the crack tip, K_I denote the mode I stress intensity factor. Function $f_{ij}(\theta)$ represent the angular distributions of the crack tip stresses. These functions can be expressed as [12]

$$\begin{Bmatrix} f_{11}(\theta) \\ f_{12}(\theta) \\ f_{22}(\theta) \end{Bmatrix} = \cos(\theta/2) \begin{Bmatrix} 1 - \sin(\theta/2)\sin(3\,\theta/2) \\ \sin(\theta/2)\cos(3\,\theta/2) \\ 1 + \sin(\theta/2)\sin(3\,\theta/2) \end{Bmatrix} \quad (4)$$

2.2. Proposed Model

The proposed model is a square plate from where the height equals the width H = B = 60mm with a thickness of t =1 mm. The problem is analysed by the Finite Element Method while using software (ABAQUS). The model is mashed per square finite element where a = b = 2 mm. The chosen material can be a composite material who is considered as an heterogeneous brittle material where the equivalent elasticity modulus is E = 70000 n/mm2 and a Poisson's ratio υ=0.2 corresponding to a glass. Taking into account boundary conditions, the proposed model is set along the Oy-axis which allows us to open the crack according to the first mode of rupture (Mode I). Using numerical analysis, the stress fields are shown in the following cartographic figures.

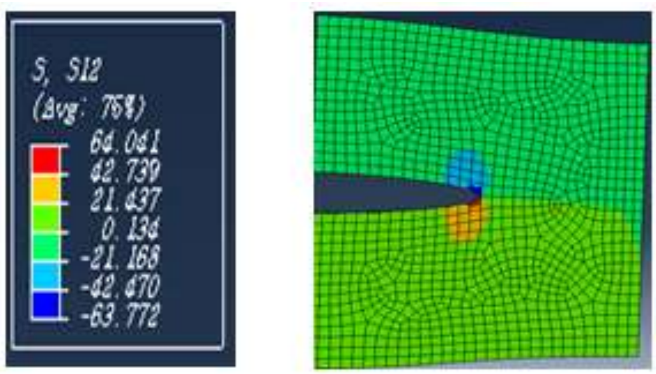

Fig. 1: Cartographic of values of stress fields σ_{12}

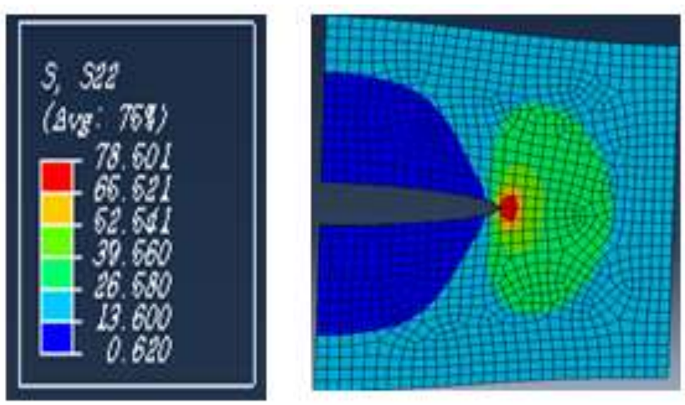

Fig. 2: Cartographic of values of stress fields stress σ_{22}

Obtained results of stress field for various crack's lengths are given in the following Table 1.

TABLE 1: Numerical results for Stress Field versus various crack's Lengths.

CRACK'S LENGTH a(mm) / STRESS (N/mm2)	0	10	20	30
σ_{11}	5.00	10.1	14.82	19.63
σ_{12}	0	26.91	46.09	64.04
σ_{22}	20	40.4	59.27	78.5

Stress fields plots for various crack's lengths are represented in the following Fig. 3.

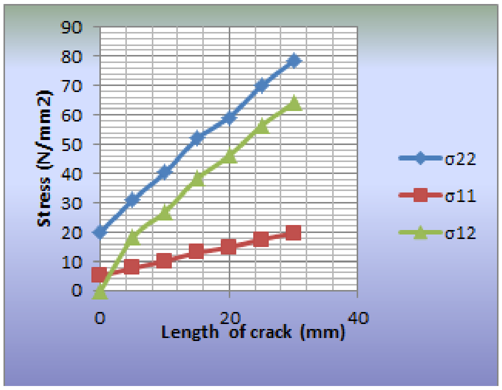

Fig. 3: Variation of stress field vs. crack's lengths.

3. Stress Intensity Factor (SIF)

Stress fields found previously are characterized by SIF. These factors are considered as essential parameters for the evaluation of the strength fracture of materials. They give information on the evolution of the crack propagation in materials through the state of the various stress fields generated during the cracking. Irwin [13] was first who considered the local stress state around a crack tip for establishing a crack propagation criterion. From then, stress fields near crack tips are divided into three basic types: Mode I (Opening), Mode II (Sliding) and Mode III (Tearing).

These SIF can be expressed by using Eq (3) and Eq (4) as follows;

$$K_I = \sigma_{yy}\sqrt{2\pi r}\ /cos(\theta/2)\{1+\sin(\theta/2)\sin(3\theta/2)\} \quad (5)$$

Stresses are concentrated at the tip of crack. For a point located at a position (r, θ) from the tip, one obtain SIF for various crack's length. These values are given in Table 2.

Obtained results and values for the Mode I stress intensity factor are shown in the following Fig. 4.

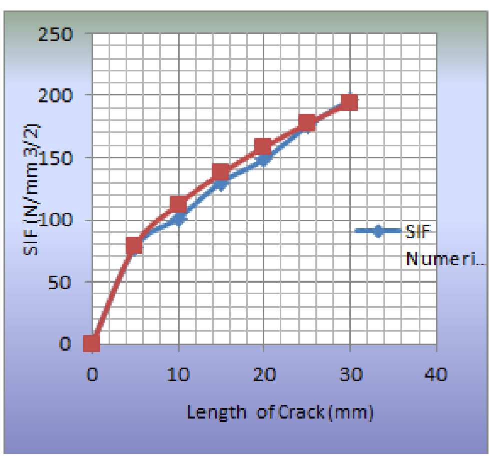

Fig. 4: Variation of (SIF) vs. Crack's lengths.

4. Energy release rates (ERR)

Energy release rate is defined as being the energy released during the propagation of the crack. For that, several researchers used the principle of the integral of Rice (J) in order to study the evolution of the crack. According to Griffith [9], the fracture is a phenomenon consuming energy and the difference occurs between the energy state of the atoms before and after cracking. The physical origin of the energy of fracture is not clearly established, but it is admitted that the phenomenon of fracture is in the expenditure of energy. Many concepts based on the released energy were proposed, but most of them are still discussed for not being satisfactory. Among the interesting results, one quotes the work of Rice [14]. Since Griffith, analysis of the phenomenon of fracture made many progress. Irwin [15] opened the way with the application of elastic solutions and thus defined the criterion of brittle fracture using some mathematical approach. From an energy point of view, analyses of both researchers Griffith and Irwin showed that, if one neglects the kinetic energy then total energy can be written as follows;

$$W_e = E_e + W_f \tag{6}$$

where E_e is the elastic energy and W_f corresponds to the energy spent during the cracking

For linearly elastic materials, J_0 can be connected to the stress intensity factor by a direct substitution of the singular solution in mode I In the case of plane stress as;

$$J_0 = K_1^2/E \tag{7}$$

According to values' of found constraints, one can calculate the rate of refund of energy at the end of the crack. The values of this energy will be recapitulated in the following Table 2:

TABLE 2: Values of Energy According to the Length of the Crack.

LENGTH OF CRACK a (mm)	0	10	20	30
J (JOULES) NUMERICAL	0	0.146	0.315	0.552
J (JOULES) THEORETICAL	0	0.179	0.358	0.538

The numerical and theoretical variation of energy release rate versus the crack's length is represented in the following Fig. 5.

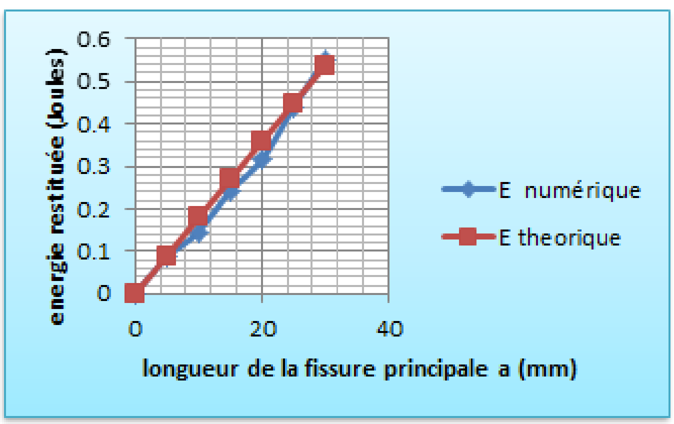

Fig. 5: Energy release rate vs. crack's lengths.

5. Discussions and Conclusion

In this research work, stress fields and stress intensity factors are determined during the propagation of a crack in a brittle material. These Stress fields and SIF's are obtained for the case of cracked models having different crack's lengths. It is shown that stress field maximum are found about the point of the crack and that for each crack's length stress field are plotted and their values are compared to the ones obtained by Irwin meaning for $\sigma_{22} > \sigma_{12} > \sigma_{11}$. Besides, one can notice that the stress fields increase at the same rate depending on the loading and the crack length. Values of stress fields found in Table 1 and curves of SIF‘s variation versus crack’s length shown in Fig. 4 are similar to those found by Irwin. It is proven also that SIF (numerical and theoretical) increase at the same rate with respect to the crack‘s length. Plotted curves are collinear in the intervals of a crack’s length a ϵ $[0, 4]$ mm and a ϵ $[24, 30]$ mm. On the other hand, for an interval of a crack’s length a ϵ $[4, 24]$, there is a small shift between points of two curves. This latest is generated by the effect to the thickness of 1mm. Then, the theoretical analysis of Irwin considers the model in 2-D meaning no thickness is taken into consideration. According to Fig. 5, the shift between the curves of energy determined numerically and theoretically depends directly on the stress intensity factor discussed previously (see Fig. 4).

6. References

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