ISBN978-93-84422-22-6
Proceedings of 2015 International Conference on Innovations in Civil and Structural Engineering (ICICSE'15)
Istanbul (Turkey), June 3-4, 2015 pp. 248-257

Analysis of Bending Behavior of Concrete Beams by the Moment-Curvature Method

Author Merimeche Fatiha[1] and Chabil Hocine[1]

[1]Department Of Civil Engineering, University Of Constantine 1

Abstract: *Solving equations of equilibrium of internal forces is very complicated due to the state of stress in steel and concrete. To determine the moment-curvature curves for reinforced concrete beams simply supported and symmetrically loaded, two methods are possible: the iterative classical method, for estimating the depth of the compression zone or the rapid method without iteration on depth values of the compression zone. The latter has allowed us to treat a case of beam section already used in our various tests.*

Keywords: *"Reinforced concrete", "Beams", "Moment-*curvature, *"Depth of compression".*

1. Introduction

The nonlinear analysis of the reinforced concrete beams by the moment curvature method allows determining the curvature of a section submitted to a given flexion. The equations of balance of the section lead to solve a system of not linear equations and the solution is obtained in a iterative way [1]. For studying our case of a beam section we used both of methods: the classic method with iteration and the quick one without iteration proposed by REZAIE 1995 [2]. The last one makes us to save a lot of time calculation.

2. Analysis by the moment-curvature method of reinforced concrete beams

2.1. Numerical and parametric study example

The beam section is 15×25cm (Fig. 1) and its characteristics are:

- Width: 15cm.
- Height: 25cm.
- Useful height: 22.5 cm.
- Height of compressed steel: 2cm.
- Embedding: 1.9 cm.
- Maximum deformation ratio of concrete: 0.0035.
- Deformation modulus of concrete: 37000 MPa.
- Concrete compressive and tensile strength f'_c and f'_t marked on each figure.
- Stretched steel section: 2.26cm^2.
- Compressed steel section 1.01 cm^2.
- Steel deformation module: 200000 MPa.
- Yield strength of steel: 400 MPa.

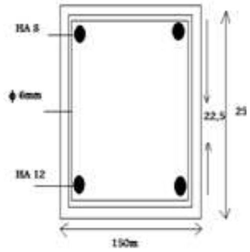

Fig. 1: "Reinforced concrete section"

2.1.1 Equilibrium equations

The static equilibrium equations of a beam cross section in bending are written

$$N = N_{int} = \int_A \sigma dA; N = \int_{v}^{v'} \sigma(y).b(y)dy \; ; M = M_{int} = \int_A \sigma y dA \; ; M = \int_{v}^{v'} \sigma(y).y.b(y)dy \qquad (1)$$

The equilibrium equations (1) are functions of two global variables, strain and depth, such as ε_s and x, two mixed variables strain and depth such as and ε'_s and $\chi = \varepsilon_c / x$

We have two variables: a strain and a depth, as for example ε_s and x.

When we choose both variables global, it is necessary to take one as pivot, and we look for the value of the other variable. In the following stage, we give a step to the fixed variable and we repeat. Normally we shall use ε_c and x as global variables. We choose the maximal strain of the compressed concrete ε_c as pivot.

The equilibrium equations of normal forces and moments:

$$N = N_c(\varepsilon_c, x) + N_S^{'}(\varepsilon_s^{'}) - N_t(\varepsilon_y, y_t) - N_S(\varepsilon_S) = 0 \qquad (2)$$

And $M = M\,\text{int}\;(\varepsilon_c, x)$,

$$M\,\text{int}(\varepsilon_c, x) = b\int_0^x f_c\,[\varepsilon_c(y)](\alpha - x + y)dy + N_s(d - d^{'}) - N_t\left(d - x - \frac{2}{3}y_t\right) \qquad (3)$$

$$\varepsilon_t = \frac{y_t}{x}\varepsilon_c \qquad \varepsilon_s = \left(\frac{d-x}{x}\right)\varepsilon_c \qquad \varepsilon_c^{'} = \left(\frac{x-d^{'}}{x}\right)\varepsilon_c \qquad (4)$$

$$y_t = h - x \text{ And } \varepsilon_t = \frac{h-x}{x}\varepsilon_c$$

These equations drive to the resolution of a system of non linear equations.

The depth x of compressed concrete is determined by the resolution of balance of the normal efforts equation (2).However, this resolution drives to a specific difficulty introduced by the state of stress into the steels which can be plasticized or not, and by the state of stress in the tense party. These states will depend on the value of x and the resolution of the problem passes by a classic iterative procedure. To solve the problem there are two methods, either classic iterative method for estimating the depth of the compression zone, or faster method REZAIE1995 [2].

The relationship of ACI [3] to describe the behavior of concrete in compression is:

$$\sigma_c(\varepsilon_c) = f_c^{'}\left[\frac{2\varepsilon_c}{0{,}002} - \left(\frac{\varepsilon_c}{0{,}002}\right)^2\right] \text{for } \varepsilon_c < \varepsilon_{cl} = 0{,}002$$

$$\sigma_c(\varepsilon_c) = f_c^{'}[1 - 300(\varepsilon_c - 0{,}002)]$$

The relationship given by BAEL 91 [4] for compression with bending is:

$$\sigma_c(\varepsilon_c) = 0{,}25\, f'_c .10^3.\varepsilon_c(4 - 10^3\varepsilon_c) \text{For } 0 \le \varepsilon_c \le 2‰$$

$$f'_c = \frac{0{,}85 \cdot f_{cj}}{\delta_b}$$

$$\sigma_c(\varepsilon_c) = f'_c \quad \text{For} 2‰ \le \varepsilon_c \le 3{,}5‰$$

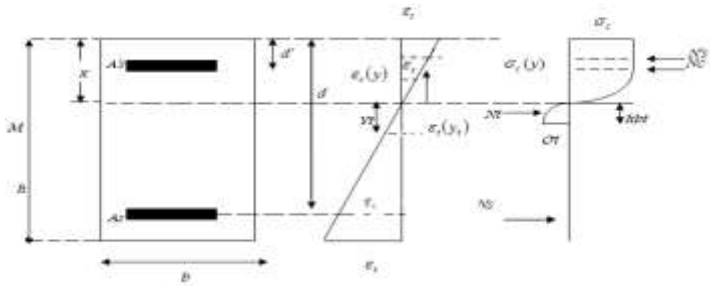

Fig. 2: "Stress – strain diagram of the section"

$\varepsilon_c = 3{,}5‰$ σ_c affected f'_c $\varepsilon_s = 3{,}5‰\,(1-\alpha)/\alpha \le 10‰$

The stress in the concrete is

$$N_b = \int_0^{4/7 X} b\left[0{,}25 f'_c (3{,}5_{y/n})(4 - 3{,}5_{y/n})\right] dy + 3b.x' f'_c/7 = 0{,}81b.x' f'_c$$

2.2. Calculation of the compressed depth x by the conventional method

-Concrete behavior law

According to A.C.I

$$N_c(\varepsilon_c, x) = \sigma(\varepsilon_c) \times (n) = b.f_{c0}^{'0{,}002}\left[\frac{\varepsilon_c}{0{,}002} - \frac{1}{3}\left(\frac{\varepsilon_c}{0{,}002}\right)^2\right]x \text{ For } \varepsilon_c < \varepsilon_{cl} = 0{,}002$$

$$N_c(\varepsilon_c, x) = \sigma(\varepsilon_c) \times (n) = b.f'_c\left[-150\varepsilon_c + 1{,}6 - \frac{0{,}038}{30\varepsilon_c}\right]x \text{ For } \varepsilon_c \ge \varepsilon_{cl} -$$

-Steel behavior law

Perfectly plastic behavior needs four assumptions:

a- The tense steels and tablets are not plasticized: $\varepsilon_s \le \varepsilon_{sl}$; $\varepsilon'_s \le \varepsilon'_{sl}$; $\sigma_s = E_s.\varepsilon_s$; $\sigma'_s = E'_s.\varepsilon'_s$

$N_s = E_s.A_s.\varepsilon_s$; $N'_s = E'_s.A'_s.\varepsilon'_s$

b- Only strained steels are plasticized: $\varepsilon_s \ge \varepsilon_{sl}$; $\varepsilon_s' < \varepsilon'_{sl}$; $\sigma_s = f_y$; $\sigma'_s = E'_s.\varepsilon'_s$

$N_s = A_s.f_y$; $N'_s = E'_s.A'_s.\varepsilon'_s$

c- Only tablets are plasticized : $\varepsilon_s < \varepsilon_{sl}$; $\varepsilon'_s \ge \varepsilon'_{sl}$; $\sigma_s = E_s.\varepsilon_s$; $\sigma'_s = f'_y$

We have $N_s = E_s.A_s.\varepsilon_s$; $N'_s = A'_s f'_y$

d- Tense steels and plastic coated tablets are: $\varepsilon_s \ge \varepsilon_{sl}$; $\varepsilon'_s \ge \varepsilon'_{sl}$; $\sigma_s = f_y$; $\sigma'_s = f'_y$

We have $N_s = A_s.f_y$; $N'_s = A'_s f'_y$

For concrete under tension:

$$N_t(\varepsilon_t, y_c) = bE_c \frac{hbt^2}{2x}\varepsilon_c = bE_c \frac{(h-x)^2}{2x}\varepsilon_c$$

To determine the compressed depth, we made a table:

$\varepsilon_c \le 0.002 \varepsilon_s \le \varepsilon_{sl}$; $\varepsilon'_s \le \varepsilon'_{sl}$

Value $x_1 + \delta x_i$ (mm)	Chosen value for ε_c	$\varepsilon_s = (\frac{d-x}{x})\varepsilon_s$	$\varepsilon'_s = \left(\frac{x-d'}{x}\right)\varepsilon_c$	$N_c(\varepsilon_c, x) = \left[bf'_c\left(\frac{\varepsilon_c}{0{,}002} - \frac{1}{3}(\frac{\varepsilon_s}{0{,}002})^2\right)\right]x$	$N_s = A'_s E'_s \varepsilon'_s$ (N)	$N_t = E_c b \varepsilon_c \frac{h_{bt}^2}{2x}$ with: $h_{tb}^2 = (h-x)^2$	$N_s = A_s E_s \varepsilon_s$ (N)

Then we see the value of X which balances the following equation:

$$N_c(\varepsilon_c, x) + N'_s = N_t(\varepsilon_t, y_t) + N_s$$

Then we redo again from 0 to 0.0019. For example, we take a step of 0.00005.Then we go for 2,3 and 4 cases we draw the same table as that of the first case but taking into account each case the stress state see (2.2.b)

Then we go to the second case when $\varepsilon_c \ge 0.002$

In this case: $N_c(\varepsilon_c, x) = \Re(\varepsilon_c) X(x) = bf'_c\left[-150\varepsilon_c + 1{,}6 - \frac{0{,}038}{30\varepsilon_c}\right]x$

Were do again the same tables by changing the formula of $N_C(\varepsilon_c, x)$

And the values of ε_c will be between 2 % and 3,5 %

In case where $\varepsilon_c \ge 0.002$; $\varepsilon_s \le \varepsilon_{sl}$; $\varepsilon'_s \le \varepsilon'_{sl}$

Value of x (mm)	$\varepsilon_c \ge 0.002$	ε_s	ε'_s	$N_c(\varepsilon_c, x) = bf'_c\left[-150\varepsilon_c + 1{,}6 - \frac{0{,}038}{30\varepsilon_c}\right]x$	$N'_s = A'_s E'_s \varepsilon'_s$	$N_t = E_c b \varepsilon_c \frac{(x - x^2)}{2x}$	$N_s = A_s E_s \varepsilon_s$	$N_c + N'_s$	$N_t + N_s$

And again the same table with another values of ε_c.

2nd case: $\varepsilon_s \ge \varepsilon_{sl}$ and $\varepsilon'_s < \varepsilon'_{sl}$

$N_s = A_s fy$; $N'_s = A'_s E'_s \varepsilon'_s$

And again for other values of ε_c between 0.002 and 0.0035

3rd case: $\varepsilon_s < \varepsilon_{sl}$ and $\varepsilon'_s \ge \varepsilon'_{sl}$

$N_s = A_s E_s \varepsilon_s$; $N'_s = A'_s f'y$

We then rebuild the same table for other values of ε_c

4thcase: $\varepsilon_s \ge \varepsilon_{sl}$ and $\varepsilon'_s \ge \varepsilon'_{sl}$

$N_s = A_s fy$; $N'_s = A'_s f'y$

And again the same table for other values of ε_c

2.2.1. Conclusion

For a value of ε_c we determine x_i and for every value of x_i we calculate the value of the deformation from the equation of compatibility. Later, we verify which calculated deformation belongs to the considered case, and confirms the value of $x=x_i$ for the value of ε_c. We go to the calculation of the moment.

If: $\varepsilon_c \le \varepsilon_{cl}$ $M\,\text{int} = bf_c^{'}x^2\left(\varepsilon_c/0{,}003 - \varepsilon_c/1{,}6.10^{-5} \right) + bf_c^{'}x(d-x)\left[\varepsilon_c/0{,}002 - \frac{(\varepsilon_c/0{,}002)^2}{3} \right] +$

$$N_s^{'}\left(d-d^{'}\right) - N_t\left(d - x - 2h_{bt}/3 \right)$$

2.3 Second method (method of possible case)

This method is proposed by REZAIE 1995 [2] and BUYLE-BODIN 1997 [5] consists in determining directly the depth of compressed zone.

- m is the number of constituents in curves of tensile steels behavior,
- n: those of the compressed steels,
- p: tense concrete

The possible number of cases of stress is equal to m*n*p and using the equation (2) we shall have m*n*p different answers.

m = 2, n = 2, p = 1

We shall have 4 possibilities

We find the same 4 hypotheses of the first method (see 2.2.b)

By four hypotheses, we can deduce the value of x (compressed depth of concrete). The ACI proposes:

$$\sigma_c(\varepsilon_c) = f_c^{'}\left[\frac{2\varepsilon_c}{0{,}002} - \left(\frac{\varepsilon_c}{0{,}002} \right)^2 \right] \text{ for } \varepsilon_c \le \varepsilon_{cl} = 0.002$$

$$\sigma_c(\varepsilon_c) = f_c^{'}\left[1 - 300(\varepsilon_c - 0{,}002) \right]$$

For $\varepsilon_c \ge \varepsilon_{cl}$

The normal effort of compression in the concrete for every interval of ε_c (and of drive respectively)

$$N(\varepsilon_c, x) = \frac{b}{x}\int_{\varepsilon_1}^{\varepsilon_2} \sigma(\varepsilon)\,d\varepsilon$$

We named: $b\int_{\varepsilon_1}^{\varepsilon_2} \sigma(\varepsilon)\,d\varepsilon$ par $\Re(\varepsilon_c)$

Thus: $N(\varepsilon_c, x) = \Re(\varepsilon_c) \times (x)$

$$N(\varepsilon_c, x) = \Re(\varepsilon_c) \times (x) = \int_0^{2^{0/00}} b\sigma_c(\varepsilon_c) d\varepsilon + \int_{2^{0/00}}^{\varepsilon_c} b\sigma_c(\varepsilon_c) d\varepsilon = b \int_0^{2^{0/00}} \left[f_c^{'} \left(\frac{2\varepsilon_c}{0,002} - \left(\frac{\varepsilon_c}{0,002} \right)^2 \right) \right] d\varepsilon + b \int_{2^{0/00}}^{\varepsilon_c} f_c^{'} [1 - 300(\varepsilon_c - 0,002)] d\varepsilon$$

$$= bf_c^{'} \left[\frac{2}{0,002} . \frac{\varepsilon_c^2}{2} - \frac{1}{3} \frac{\varepsilon_c^3}{(0.002)^2} \right] + \left[\varepsilon_s - 300 \frac{\varepsilon_c^2}{2} + (0,6)^{\varepsilon_C} - c \right]$$

$$N(\varepsilon_c, x) = \Re(\varepsilon_c) \times (x) = bf_c^{'} \left[\frac{\varepsilon_c^2}{0,002} - \frac{1}{3} \frac{\varepsilon_c^3}{(0,002)^2} \right] + \left[-150\varepsilon_c^2 + 1,6\varepsilon_c - c \right]$$

If we divide on ε_c, we shall have:

$$N(\varepsilon_c, x) = \Re(\varepsilon_c) \times (x) = bf_c^{'} \left[\frac{\varepsilon_c}{0,002} - \frac{1}{3} \left(\frac{\varepsilon_c}{0,002} \right)^2 \right] \text{for } \varepsilon_c < \varepsilon_{cl} = 0,002$$

$$N(\varepsilon_c, x) = \Re(\varepsilon_c) \times (x) = bf_c^{'} \left[-150\varepsilon_c + 1,6 - \frac{0,0038}{30\varepsilon_c} \right]$$

$$bf_c^{'} \left[\frac{\varepsilon_c}{0,002} - \frac{1}{3} \left(\frac{\varepsilon_c}{0,002} \right)^2 \right] = bf_c^{'} \left[-150\varepsilon_c + 1,6 - \frac{0,0038}{30\varepsilon_c} \right]$$

We are going to replace ε_c with 0,002, we shall have:

$$bf_c^{'} \left[\frac{0,002}{0,002} - \frac{1}{3} \left(\frac{0,002}{0,002} \right)^2 \right] = bf_c^{'} \left[-150(0,002) + 1,6 - \frac{c}{0,002} \right]$$

$$\left[1 - \frac{1}{3} \right] = \left[-0,3 + 1,6 - \frac{c}{0,002} \right]$$

$$\frac{2}{3} = [1,3 - \frac{c}{0,002}]$$

Thus $1,3 - \frac{2}{3} = \frac{c}{0,002} \Rightarrow \frac{1,9}{3} = \frac{c}{0,002} \Rightarrow C = \frac{1,9 \times 0,002}{3} = \frac{0,0038}{3}$

$$N(\varepsilon_c, x) = \Re(\varepsilon_c) \times (x) = \begin{cases} \Re(\varepsilon_c) = bf_c^{'} \left[\frac{\varepsilon_c}{0,002} - \frac{1}{3} \left(\frac{\varepsilon_c}{0,002} \right)^2 \right] \\ \Re(\varepsilon_c) = bf_c^{'} \left[-150\varepsilon_c + 1,6 - \frac{0,038}{30\varepsilon_c} \right] \end{cases}$$

We consider now the state of strain of the tensile concrete:

$$N_t(\varepsilon_c, y_t) = \frac{1}{2} b h_{bt}.\varepsilon_t E_c \text{ with } \varepsilon_t : \frac{\varepsilon_c}{x} = \frac{\varepsilon_t}{h_{bt}} \Rightarrow \varepsilon_t = \frac{\varepsilon_c . h_{bt}}{x}$$

The equations of balance of the normal efforts and the moments of the section respectively are:

$$\varepsilon_s = \left(\frac{d-x}{x}\right)\varepsilon_c \; ; \qquad \varepsilon_s^{'} = \left(\frac{x-d^{'}}{x}\right)\varepsilon_c \; ; \qquad \varepsilon_t = \frac{y_t}{x}\varepsilon_c$$

$$N = N_c(\varepsilon_c, x) + N_s^{'}(\varepsilon_s^{'}) - N_t(\varepsilon_t, y_t) - N_s(\varepsilon_s) = 0$$

$$y_t = h - x \Rightarrow \varepsilon_t = \left(\frac{h-x}{x}\right)\varepsilon_c$$

We have 4 possible cases for a maximal deformation of the compressed concrete fixed εc. The value of the compressed depth is chosen among the four values calculated according to the equation (2).

1st hypothesis: $\varepsilon_s < \varepsilon_{sl}$,$\varepsilon'_s < \varepsilon'_{sl}$ the tense and compressed armatures are not plasticized

$$N(\varepsilon_c, x) = \Re(\varepsilon_c) \times (x)$$

$$\Re(\varepsilon_c) = bf_c^{'}\left[\frac{\varepsilon_c}{0{,}002} - \frac{1}{3}\left(\frac{\varepsilon_c}{0{,}002}\right)^2\right] \quad \varepsilon_c \le \varepsilon_{cl} = 0{,}002$$

$$\Re(\varepsilon_c) = bf_c^{'}\left(-150\varepsilon_c + 1{,}6 - \frac{0{,}038}{30\varepsilon_c}\right) \rightarrow \varepsilon_c > \varepsilon_{cl} = 0{,}002$$

$$X(x) = x$$

$$N_t(\varepsilon_c, x) = bE_c \frac{h_{bt}^2 \varepsilon_c}{2x}$$

$$\Re(\varepsilon_c) X(x) + \begin{cases} A_s^{'} E_s^{'} \varepsilon_s^{'} \rightarrow si: f_s^{'} \le f^{'}y \\ A^{'} f^{'}y \end{cases} = bE_c \frac{h_{tb}^2 \varepsilon_c}{2x} + \begin{cases} A_s E_s \varepsilon_s \rightarrow si: f_s \le fy \\ A_s fy \end{cases}$$

$$\text{N} = \Re(\varepsilon_c)x + A_s^{'} E_s^{'} \varepsilon_s^{'} - A_s E_s \varepsilon_s - bE_c \frac{h_{tb}^2}{2x}.\varepsilon_c = 0$$

If we neglect the tense concrete, we shall have:

$$\text{N} = \Re(\varepsilon_c)x + A_s^{'} E_s^{'} \varepsilon_s^{'} - A_s E_s \varepsilon_s = 0$$

$$\Re(\varepsilon_c)x + A_s^{'} E_s^{'}\left(\frac{x-d'}{x}\right)\varepsilon_c - A_s E_s\left(\frac{d-x}{x}\right)\varepsilon_c = 0$$

$$\Re(\varepsilon_c)x^2 + A_s^{'} E_s^{'}(x-d')\varepsilon_c - A_s E_s(d-x)\varepsilon_c = 0$$

$$\Re(\varepsilon_c)x^2 + A_s^{'} E_s^{'} x\varepsilon_c - A_s^{'} E_s^{'} d'\varepsilon_c - A_s E_s d\varepsilon_c + A_s E_s x\varepsilon_c = 0$$

$$\Re(\varepsilon_c)x^2 + [(A_s^{'} E_s^{'} + A_s E_s)x\varepsilon_c] - (A_s^{'} E_s^{'} d' + A_s E_s d)\varepsilon_c = 0$$

The solution of this equation of the 2nd degree is:

$$\Delta = [\varepsilon_c(A_s^{'}E_s^{'} + A_s E_s)^2] + 4\Re\varepsilon_c(A_s^{'}E_s^{'}d' + A_s E_s d)$$

$$\sqrt{\Delta} = [\varepsilon_c(A_s^{'}E_s^{'} + A_s E_s)^2 + 4\Re\varepsilon_c(A_s^{'}E_s^{'}d' + A_s E_s d)]^{1/2}$$

$$x_1 = \frac{-\varepsilon_c(A_s E_s + A_s^{'}E_s^{'}) \pm \sqrt{\Delta}}{2\Re} \Rightarrow \sqrt{\Delta} = radical\,\varepsilon_c$$

$$x_1 = \frac{radical - \varepsilon_c(A_s E_s + A_s^{'}E_s^{'})}{2\Re} \text{ if}$$

$$\varepsilon_s = \frac{d - x_1}{x_1}\varepsilon_c \le \varepsilon_l \;\; \varepsilon_s^{'} = \frac{x_1 - d'}{x_1}\varepsilon_c \le \varepsilon_l^{'}$$

$x = x_1$ Otherwise the following hypothesis

The second hypothesis: only tense steels are plasticized

$(\varepsilon_s > \varepsilon_l \;, \varepsilon_s^{'} < \varepsilon_l^{'})$

$$\Re(\varepsilon_c)x + A_s^{'}E_s^{'}\left(\frac{x - d'}{x}\right)\varepsilon_c - A_s fy = 0$$

$$\Re(\varepsilon_c)x^2 + A_s^{'}E_s^{'}x\varepsilon_c - A_s^{'}E_s^{'}d'\varepsilon_c - A_s fyx = 0$$

$$\Re(\varepsilon_c)x^2 + \left(A_s^{'}E_s^{'}\varepsilon_c - A_s fy\right)x - A_s^{'}E_s^{'}d'\varepsilon_c = 0$$

The solution of this equation is:

$$x_2 = \frac{-(\varepsilon_c A_s' E'_s - A_s fy) \pm \sqrt{\Delta}}{2\Re}$$

$$x_2 = \frac{radical - (A_s^{'}E_s^{'}\varepsilon_c - A_s fy)}{2\Re} \text{ if :}$$

$$\varepsilon_s = \frac{d - x_2}{x_2}\varepsilon_c \ge \varepsilon_l \;\; \varepsilon_s^{'} = \frac{x_2 - d'}{x_2}\varepsilon_c \le \varepsilon_l$$

$$x = x_2$$

Otherwise the following hypothesis the third hypothesis: Only compressed steels are plasticized

$(\varepsilon_s < \varepsilon_l \;, \varepsilon_s^{'} > \varepsilon_l)$

$$\Re(\varepsilon_c)x + A_s^{'}f^{'}y - A_s E_s \varepsilon_s = 0$$

$$\Re(\varepsilon_c)x + A_s^{'}f^{'}y - A_s E_s\left(\frac{d - x}{x}\right)\varepsilon_c = 0$$

$$\Re(\varepsilon_c)x^2 + A_s^{'}f^{'}y.x - A_s E_s(d - x)\varepsilon_c = 0$$

$$\Re(\varepsilon_c)x^2 + A_s^{'}f^{'}y.x + A_s E_s x\varepsilon_c - A_s E_s d\varepsilon_c = 0$$

$$\Re(\varepsilon_c)x^2 + \left[\varepsilon_c(A_s E_s) + A_s^{'} f^{'} y\right]x - A_s E_s d\varepsilon_c = 0$$

$$x_3 = \frac{radical(\varepsilon_c) - (A_s E_s \varepsilon_c + A_s^{'} f^{'} y)}{2\Re}$$ if

$$\varepsilon_s = \frac{d - x_3}{x_3}\varepsilon_c \leq \varepsilon_l \; \varepsilon_s^{'} = \frac{x_3 - d'}{x_3}\varepsilon_c \geq \varepsilon_l$$

$x = x_3$

Otherwise the following hypothesis

The fourth hypothesis: Tense and compressed steels are plasticized

$N_s = A_s f_y$; $N'_s = A'_s f'_y$; if

$$\varepsilon_s^{'} = \frac{x_4 - d'}{x_4}\varepsilon_c \geq \varepsilon_l^{'} \; \varepsilon_s = \frac{d - x4}{x4}\varepsilon_c \geq \varepsilon_l$$

$$x = x_4$$

If $\varepsilon_c \leq \varepsilon_{cl}$

Mint=$b.f'_y.x^2(\varepsilon_c/0.003-\varepsilon_c^2/1.6.10^{-5})+b.f'_y.x(d-x)[\varepsilon_c/0.002-(\varepsilon_c/0.002)^2/3]+N'_s(d-d')-N_t(d-x-2h_{bt}/3)$

If $\varepsilon_c > \varepsilon_{cl}$

Mint=$b.f'_y(x/\varepsilon_c)^2(-100\varepsilon_c^3+0.8\varepsilon_c^2-2.2* \; 10^{-5} \; /30)+b.f'_y.x/\varepsilon_c(d-x)[-150\varepsilon_c^2+1.6\varepsilon_c-0.038/30]+N'_s(d-d')-N_t(d-x-2h_{bt}/3)$.

The curvature is: $1/r(\varepsilon_c,x) = \varepsilon_c/x$

The fracture criteria's are the fracture of compression's zone of concrete ($\varepsilon_c \geq 0,0035$) or the fracture of steel ($\varepsilon_s \geq 10‰$).

2.3.1 Conclusion

This method of possible cases allows us to calculate the exact value of x, without making of iteration as for the case of the classic method, what makes us save a lot of time of calculation. This method facilitates us to determine the curvature of rectangular reinforced concrete subjected to simple bending.

The made work allowed us to trace curves moment-curvature and the variation of the depth of the compression zone.

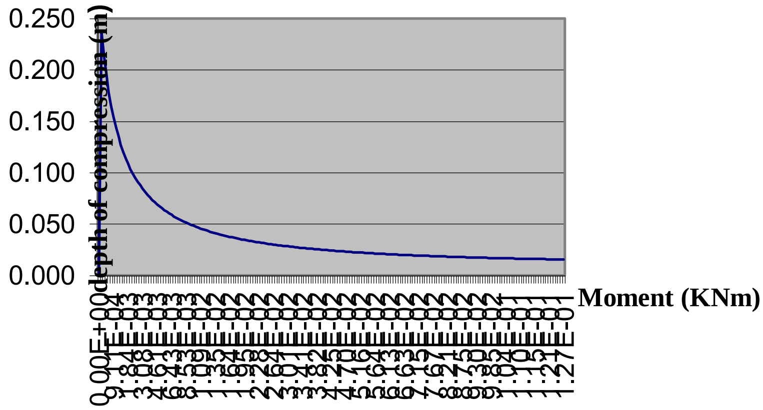

Fig. 3: "variation of the depth of the zone of compression"

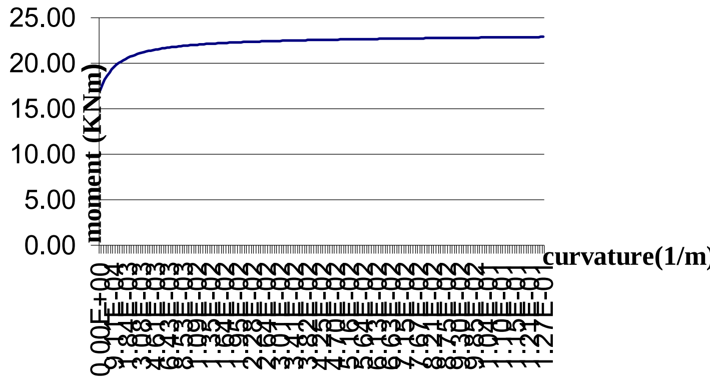

Fig. 4: "moment-curvature"

3. References

[1] FREYDOON REZAIE : « Etude du comportement en flexion de poutres en béton armé corrodé : « Modélisation moment-courbure modifiée prenant en compte le raidissement en traction du béton fissuré » juin 1999

[2] REZAIE. F., 1995 « Détermination de la courbure de flexion statique d'une section en béton armé en recherche directe et indirecte « Mémoire de DEA – Groupe de formation Doctorale, Université de Lille- Université d'Artois- Ecole Centrale de Lille.

[3] R.PARK et T PAULAY.," Reinforced concrete structures" department of civil Engineering, university of Canterbury, Christchurch, New Zealand. P 203.

[4] B.A.E.L. 1993 « Béton armé aux états limites » DTU, 1993

[5] REZAIE F., BUYLE-BODIN, 1997 « Recherche de la courbure de poutres en flexion simulation par des modèles de types tirant » 4ème réunion annuelle du réseau GEO, Aussois, France, 1997.