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The Delaunay Diagram of Roundabout Cairo since 1867

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Abstract**:** *The graph of Delaunay Diagram integrates the geometrical measures with syntactic and algorithmic properties to determine the spatial structure of Haussmannized Cairo in 1867 for conservation. The essence of dense circumcircle property stretches from 'Tahrir' to 'Ezbekia Garden' with boulevard projection towards Cairo monuments such as the Pyramids. Larger circumcircles span Cairo in overlap between the old and new towns. The circumcircle structure ties with the various geometrics in dynamic clustering of interval values that zone-up or scatter over the point pattern distribution. The extreme interval ranges correlate with the two poles of the 'Citadel' and 'Cairo Station' in contrast to the central corridor. Meanwhile, the major public facilities and the royal palaces optimize the spatial integrity and control of the Delaunay structure, with the traversable property of graph vertices in maximal matching between the graph degree and radial boulevards. The absolute correlation with the Delaunay dimensions observes 'Tahrir' morphology of intermediary regenerative structure.*

Keywords: *Cairo, Roundabout, Delaunay Diagram, Spatial Structure*

1. Introduction

The Delaunay diagram represents the dual geometry of Voronoi diagram. The Delaunay tracing connects between all the pairs of generator points having a shared Voronoi edge. The generator points are the centroids of spatial convex hull corresponding to the exclusive selection of typological space for morphological analysis [4]. The Delaunay methodology is used to extract the spatial structure of the generative convex hull, with extended properties of Graph Theory algorithms [for example 6 & 9]. The Delaunay concept determines the system retrieval of built-up spatial structure and also conceptualizes the generation of design alternatives for comparative selection with geometrical proofs. The Delaunay realm parallels the space syntax method of systematic approach towards the social logic of space, though in different processing. The space syntax subdivides the void of built-up settlements into convex mapping with the conversion to graph representation of various algorithmic properties to deduce or generate the logic of designing the spatial structure [5]. In this regard, the proposed study attempts to integrate the two methods through their complementary graph algorithms for an extended scope of spatial structure with pilot application on Cairo. In this respect, the Haussmannization of Cairo in 1867, which planned roundabout plazas with boulevard connections, is selected for the Delaunay application with the space syntax inputs to explore the spatial structure of roundabout Cairo for future planning.

The Egyptian Royalty invited the French engineer, Haussmann, whom in 1867 designed the regular network of 15-roundabouts in contrast to the irregular pattern of old Cairo, with low density buildings in huge gardens [3]. The scheme was part of the ceremonial event to inaugurate the Suez Canal navigation, which was attended by many monarchies of the world [1]. The plan was realized in 1874 under the direction of the other French engineer, Le Grand, with the resulted survey map of Cairo "Fig. 1". The roundabout plazas are characterized by prismatic blocks of neo-classical building façades for various public, commercial, office and residential functions, with the development of Chateaux housing towards the outskirts of Cairo on the 'garden city' concept. Later the British occupation of Egypt in 1882 continued the roundabout structure of Haussmannized Cairo, which extended radial boulevards to the suburbs of integration with the rural peripheries. This planning practice of Cairo, however, experienced another phase of national independence in 1952 with cultural changes that affected the further development of roundabout spatial structure. The core Haussmannization plan of Cairo at present corresponds to Downtown Cairo with different redevelopment of buildings and open spaces [2]. Some researchers at present conduct conservational studies on the special urban heritage of downtown Cairo, with the most extensive documentation of, M. Scharabi, detailing the urban and building characteristics of the area [7]. Also the continual studies of, M. Volait, documents the special urbanism with the socioeconomic structure of downtown Cairo for conservation [for example 8]. On the other, the Egyptian Urban Planning Department in association with Cairo University and Cairo Governorate continue the efforts of conserving the area against the deteriorated image [11]. Meanwhile, the topic has gained memorial dimension due to Haussmann's core roundabout of 'Tahrir' that spotted the nationwide up-rise with symbolic characteristics for future developments [10]. Further scope of this study attempts to qualify the urban heritage of roundabout plazas in downtown Cairo through the conceptual structure

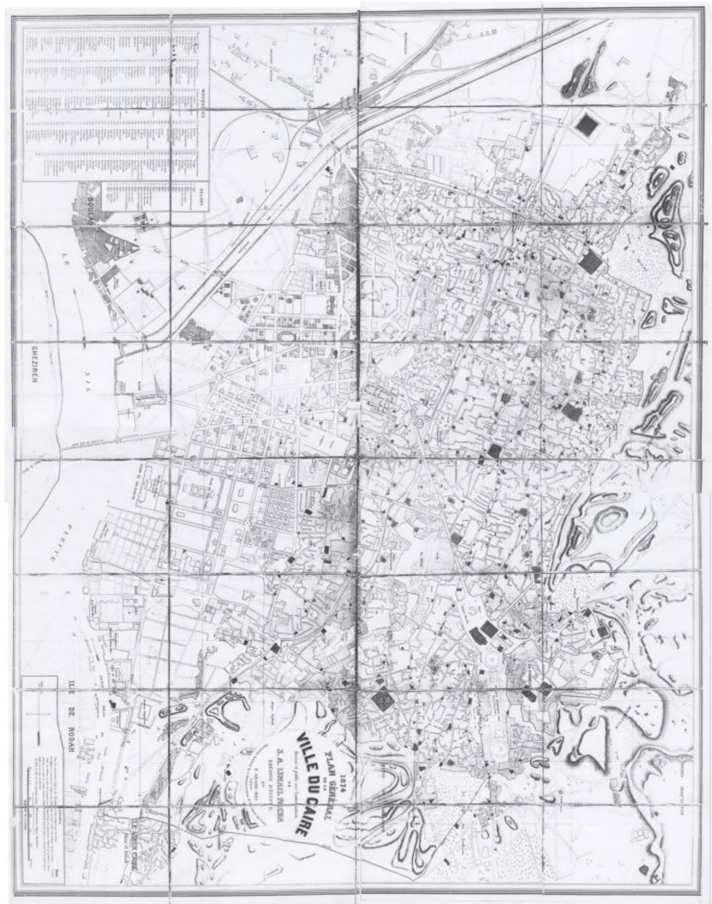

Fig. 1: Haussmannized Cairo since 1867

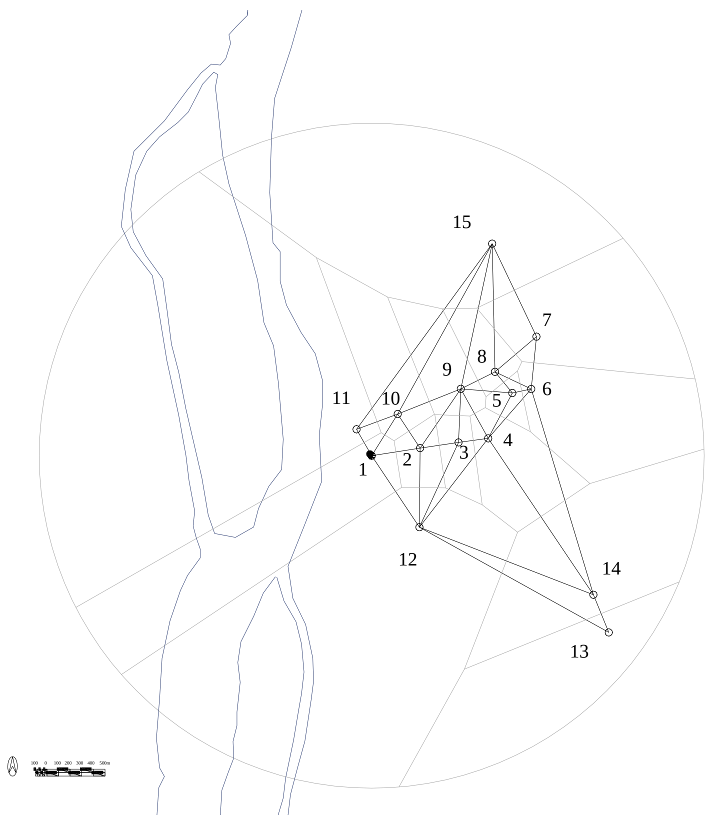

Fig. 2: The Delaunay Diagram of Roundabout Cairo

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of Delaunay graph algorithms, with the extended scope of space syntax to determine the logic of spatial configuration for future conservation.

2. Delaunay Diagram Analysis

The Delaunay diagram can be traced over the Voronoi tessellation of generator points "Fig. 2". In this process, the initial step is locating the generator points in the Euclidean plane, which in this case the roundabout plazas of Haussmann's Cairo. Each roundabout is represented as a generator point at the center of the roundabout itself. The next step is to bisect the lines joining between the adjacent generator points, thus the Voronoi diagram of spatial tessellation. Based on the Voronoi tessellation, the Delaunay diagram joins between any two generative points across their shared Voronoi edge if existing. The resultant Delaunay diagram forms triangles of connected vertices corresponding to the roundabout generators, while the underneath Voronoi diagram defines the territories of each generator point or roundabout at the center. Each of the two interrelated but different diagrams applies analytical criteria to determine the structure of space. On one hand, the Delaunay diagram forms a geometrical graph of algorithmic properties for the spatial configuration in question. On the other, the Voronoi diagram provides nonempty polygons in comparison of spatial structure, which includes the observation of land use, street pattern, building heights and other urban form characteristics. The former Delaunay diagram is privileged by the geometric graph, which considers the distance factor in the analysis of algorithmic determination for the spatial structure of generator points instead of the not-to-scale topological graph structure. The latter Voronoi diagram, however, constructs a convex hull that extends to infinity with universal scopes of multidimensional spatial analysis for the generator points as well. The concerned Delaunay analysis of this study observes the geometrical dimensions of the Delaunay graph with extended properties of Graph Theory algorithms, in addition to the space syntax of spatial integration and control measures of the topological graph representation "Table 1".

TABLE 1: The Geometrical Measurements of Delaunay Diagram

Ser.	Triangle	Area	Perim-eter	A/P ratio	L1	L2	L3	Mean-L	A1	A2	A3	Per-1	Per-2	Per-3	Mean-Per	Circum-circle
1	(1,10,11)	25381.08	765.97	33.13	189.18	272.01	304.78	255.32	37	62	81	94.59	136	152.3	127.66	308.96
2	(1,2,10)	34983.34	861.4	40.61	252.67	303.96	304.78	287.14	49	65	66	126.3	151.9	152.3	143.56	334.54
3	(1,2,12)	73571.69	1327.6	55.41	303.96	490.9	532.78	442.55	35	65	80	151.9	245.4	266.3	221.27	540.26
4	(2,9,10)	52157.88	1119.8	46.58	252.67	421.48	445.65	373.27	34	68	78	126.3	210.7	222.8	186.63	454.96
5	(2,3,9)	39450.88	1019.7	38.68	241.15	332.91	445.65	339.90	32	47	101	120.5	166.4	222.8	169.95	453.43
6	(2,3,12)	58494.79	1311.1	44.61	241.15	490.9	579.05	437.03	24	57	99	120.5	245.4	289.5	218.51	585.92
7	(3,4,9)	30421.5	869.9	34.97	185.7	332.91	351.29	289.97	31	69	80	92.85	166.4	175.6	144.98	356.94
8	(3,4,12)	45262.41	1461.7	30.96	185.7	579.05	697.03	487.26	13	44	123	92.85	289.5	348.5	243.63	827.95
9	(4,5,9)	46949.85	990.99	47.37	318.73	320.96	351.29	330.33	56	57	67	159.3	160.4	175.6	165.16	382.72
10	(4,5,6)	14613.23	844.3	17.30	119.59	318.73	405.98	281.43	13	37	130	59.79	159.3	202.9	140.71	529.46
11	(4,6,14)	229217.7	2908.2	78.81	405.98	1169	1333.1	969.41	17	58	105	202.9	584.5	666.5	484.7	1380.2
12	(4,12,14)	386855.9	3024.1	127.9	697.03	1158	1169.0	1008	35	72	73	348.5	579	584.5	504	1219.6
13	(5,8,9)	19623.94	727.82	26.96	170.03	236.83	320.96	242.61	31	46	103	85.01	118.4	160.4	121.3	329.31
14	(5,6,8)	9021.05	538.55	16.75	119.59	170.03	248.93	179.52	26	37	117	59.79	85	124.4	89.75	280.54
15	(6,7,8)	38296.93	913.32	41.93	248.93	326.66	337.73	304.44	44	66	70	124.4	163.3	168.8	152.2	358.55
16	(7,8,15)	104317.8	1771.7	58.87	337.73	638.87	795.16	590.59	24	51	105	168.8	319.4	397.5	295.2	822.34
17	(8,9,15)	85186.02	1953.3	43.61	236.83	795.16	921.31	651.10	14	51	115	118.4	397.5	460.6	325.5	1018.3
18	(9,10,15)	161011.2	2551.3	63.1	421.48	921.31	1208.5	850.43	17	39	124	210.7	460.6	604.2	425.2	1457.2
19	(10,11,15)	107092.1	2906.4	36.84	272.01	1208.5	1425.9	968.81	7	34	139	136	604.2	712.9	484.4	2188.4
20	(12,13,14)	106244.3	2754.3	38.57	252.8	1158	1343.9	918.26	8	39	133	126.4	579.0	671.9	459.1	1848.3
	Average	83407.69	1481.0	46.15	272.64	582.3	676.14	510.37	27	53.2	99.4	136.3	291.1	338	255.1	783.91

L: Length of Delaunay edge A: Angle of Delaunay triangles Per: Perpendicular distance from vertex to bisecting edge

The Delaunay diagram of roundabout Cairo forms a geometrical graph of 20-triangles with detailed distance and angular measures of the generator points. The circumcircle property is the circle connecting between each three vertices (triangle) of the Delaunay graph, which reflects the spaced distribution of the generator points

"Fig. 3". The wide variance of circumcircle diametric measure indicates the random distribution of roundabout generators with specific point process for each location. The largest circumcircle belongs to generators (10, 11, 15) that extends over the Nile, while the smallest one of (5, 6, 8) borders between the new European Quarter and old Cairo where the socio-economic hub of 'Ezbekia Garden' is located. It symbolizes the opening of the roundabout structure towards the Nile versus the filteration process of closest generators at the overlap of diverse Cairo. The clustering of the circumcircle measurements set into the largest range with over 1km in diameter of triangles '19, 20, 18, 11, 12, 17', average 1~0.5km of triangles '8, 16, 6, 3, 10', and smallest range of less than 0.5km of triangles '4, 5, 9, 15, 7, 2, 13, 1, 4'. The range of distant vertices has the most encounters of '13, 14, 15' generators, which locate at the opposite poles along the north-south axis of Cairo. The average range is more attracted towards generator-12, which bypasses central Cairo with open lands along the Nile. Meanwhile, the shortest circumcircles extend along the east-west axis from the Nile edge up to 'Ezbekia Garden' with further extension eastwards through 'Boulevard Neuve' where 'El-Azhar' historical monument is reached. The zigzagging roundabouts with compact circumcircles form a narrow longitudinal corridor in line with the underlying Voronoi edges of generator-1 that corresponds to 'Tahrir' roundabout. Nevertheless, the infinite Voronoi polygon of 'Tahrir' projects between 'Gezira' and 'Roda' islands with unobstructed breadth over the Nile towards the Giza suburb where the Pyramids are located. Thus, the Delaunay structure of dense axial circumcircles matches with the Voronoi of 'Tahrir' that targets the symbols of Egypt. This is evident by the roundabout of 'Tahrir' having the earliest bridge of 'Kasr El Nile' in realized structure. Overview of the circumcircle dimension expands the point distribution to span the whole of Cairo and not only the new European Quarter, with spinal corridor of infinite projection to integrate the separate monuments.

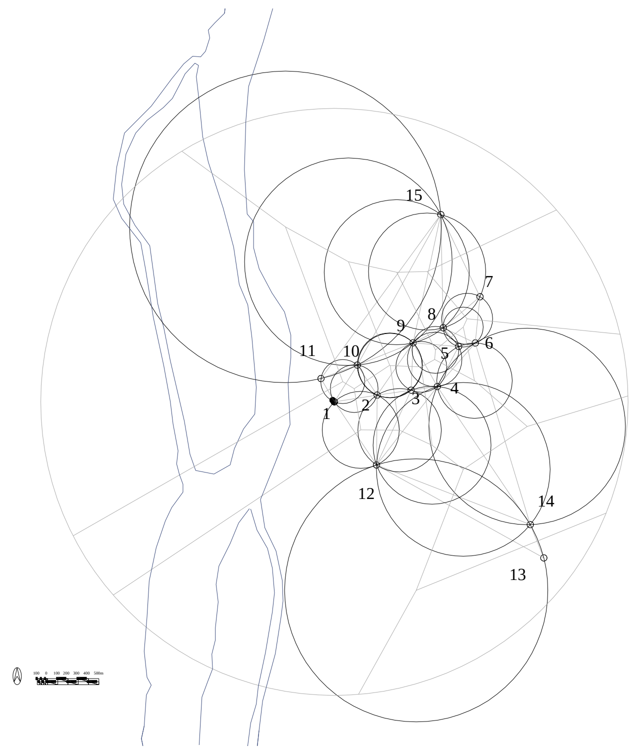

Fig. 3: The Circumcircle of Delaunay Diagram

The other Delaunay measures include the triangular area and perimeter with their ratio, length of tri-sides, angles and the perpendicular distance from each vertex to its bisecting edge of adjacency. The minimum angles cluster into over 35-degrees of triangles '9, 2, 15, 1, 3, 12', 20~35◦ of '4, 5, 7, 13, 14, 6, 16', and less than 20◦ of '11, 18, 17, 8, 10, 20, 19' in sequence. The higher range corresponds to the triangles at the two poles of 'Tahrir' and 'Ezbekia Garden' with the less range occupying the zone in between, while the least angles encounter the other two opposite poles of 'Cairo Station' and 'Cairo Citadel at generators '14 & 15' respectively. Therefore, the skewed angles correlate with the large circumcircles in contrast to the more obtuse of smaller circles. The middle range, nevertheless, re-cluster into over 60-degrees of '12, 7, 4, 15, 2, 3, 1', 45~60◦ of '11, 6, 9, 16, 17, 5, 13', and less than 45◦ of '8, 18, 20, 10, 14, 19'. The large mix of locations among the three ranges indicates a more scattered pattern of triangulation all through the generator points. The distinctive observation, however, is the total encounters of 'Tahrir' triangles at the first range of widest middle angles, while all of the other generators have different encounters among the three ranges. The widest angles of more than 120-degrees form a cluster of triangles '19, 20, 10, 18, 8', followed by 90~120◦ of '14, 17, 11, 16, 13, 5, 6', and less than 90◦ of '1, 3, 7, 4, 12, 15, 9, 2'. These maximum ranges correspond to the reverse of the minimum angular counterparts where the minimum lower ones correspond to the maximum higher and vice versa. Accordingly, the poles of 'Tahrir' and 'Ezbekia' sustain the maximum lower angles to keep moderate, while the extreme widest and narrowest angles accumulate at 'Cairo Station' and 'Cairo Citadel' triangles. The latter, however, observes more network of generator set '13, 14, 12, 4, 6' of almost collinear distribution among each other, versus the single structure of generator-15. The most stable angular distribution is 'Tahrir' of equilateral minimum-upper and maximum-lower to keep within the moderate triangular measures.

The length of triangular sides compares with the angular structure of Delaunay diagram. The shortest triangular sides cluster into longer lengths over 300m of '12, 8, 11, 16, 9, 3', medium 200~300m of '19, 20, 2, 4, 15, 5, 6, 17', and less than 200m of '1, 7, 8, 13, 10, 14'. The longer range is distributed among the variously sized circumcircles of widening up and narrowing down angular measures with the most of encounters in generto-4 triangles of different sizes. The shortest-medium range follows suit, but with more concentration at generator-2 triangles of smaller sized circumcircles. The least range, however, has the most correlation with the small-sized core circumcircles of moderate angles. The observation is the equal distribution of 'Tahrir' triangles with one triangle per group in contrast to the previous measures of absolute clustering in single group. The medium lengths rearrange into clusters of longer range over 700m of triangles '19, 11, 12, 20, 18, 17', 400~700m of '16, 8, 3, 6, 4', and shorter than 400m of '5, 7,15, 9, 10, 2, 1, 13, 14' lengths. The longer range clearly correlates with the largest circumcircles, while the middle range intermixes among the different sized measures, but the shorter range re-correlates with the smaller circumcircles of moderate triangulation. Once again 'Tahrir' invests more in proportion to the shorter-length correlation of extended group, while also contributes by one triangle in the fewer middle range of random geometrical structure. Meanwhile, the longest lengths re-cluster into longer than 1000m of triangles '19, 20, 11, 18, 12', 500~1000m of '17, 16, 8, 6, 3', and shorter than 500m of '4, 5, 10, 7, 9, 15, 13, 1, 2, 14'. The longest range enforces the correlation with the skewed measures of largest circumcircles and widest-narrowest angles. The medium lengths, nevertheless, concentrate at the two generators '12 & 15' of farthest connectivity to the poles of 'Tahrir & Ezbekia Garden' respectively. The shortest range re-correlates with the smallest circumcircles. The specific 'Tahrir' considers the farthest connectivity of graph with one triangle, while normalizes in the shortest range by its remaining two triangles.

The perpendicular distance from each Delaunay vertex to the bisecting edge of the dual Voronoi diagram explores another dimension of the triangulated geometrical graph. The shortest distances cluster into longer than 150m of triangles '12, 18, 11, 16, 9, 2', 100~150m of '19, 20, 2, 4, 15, 5, 6, 17', and shorter than 100m of '1,7, 8, 13, 10, 14'. While the longest and shortest ranges positively correlate with the size of circumcircles, the medium range largely mixes from spaced to narrowed point distribution. The triangles of 'Tahrir' followed suit by varied distribution to match with the shortest three ranges in separate. The medium range of distances re-clustered into longer than 300m of '19, 11, 12, 18, 17, 16', 150~300m of '8, 3, 6, 4, 5, 7, 15, 9, 10, 2', and shorter than 150m of '1, 13, 14, 20'. The three ranges reset in corollary to the point distribution of circumcircles, except only the elongated triangle-20 in the shortest range. Also 'Tahrir' has slightly shifted one of its triangles from the bottom medium to the top shortest range, thus the latter range becoming of single representative from all three ranges. It suggests the geometrical tendency to dissolve the distant classification with more homogenous point distribution, especially at the poles of interest. The longest distances, nevertheless, re-cluster into longer than 500m of '19, 20, 11, 18, 12', medium 200~500m of '17, 16, 8, 6, 3, 4, 5, 10', and shorter than 200m of '7, 9, 15, 13, 1, 2, 14'. The top and bottom ranges correlate with the point pattern, but not the mix of longest-medium range with one of 'Tahrir' triangles included. Overview of perpendicular distances generates dynamic clustering of systematic against random processes of interactive Delaunay geometry, especially for the adjustable 'Tahrir' structure.

The area of Delaunay diagram clusters into triangles '12, 11, 18, 19, 20, 16' of more than 100,000m^2, followed by '17, 3, 6, 4, 9, 8' triangles of 40,000~100,000m^2, and '5, 15, 2, 7, 1, 13, 10, 14' triangles of less than 40,000m^2. The expected correlation of areas with circumcircles is maintained throughout the three groups, though of extreme changes in sequence. For example, the largest circumcircle of (10, 11, 15) is different from the largest area of (4, 12, 14) triangle, even though of their same belonging to largest groups of both criteria. Meanwhile, the smallest circumcircle of (5, 6, 8) is itself the smallest triangular area to prove constant. In this regard, the pattern of point distribution inter-varies within the stable clustering of geometrical dimensions. This gives the flexibility of roundabout structure at random, while retrieving the spatial structure of each through the shape of comparable geometrics. Nevertheless, the perimeter measure clusters into longest range with over

1500m of triangles ‘12, 11, 19, 20, 18, 17, 16’, then 1000~1500m of ‘8, 3, 6, 4, 5’, and the shortest of less than 1000m of ‘9, 15, 7, 2, 10, 1, 13, 14’ in sequence. Compared to area, the perimeter favors more the sequenced correlation with the circumcircle measure, while in the meantime preserves the narrowest variance from the area measure. Thus, the perimeter intermediates between circumcircle and area differences in order to synchronize their shaping with comparable clusters. The ratio of area to perimeter clusters into more than 50-ratio of ’12, 11, 18, 16, 3’, followed by 40~50-ratio of ‘9, 4, 6, 17, 15, 2’, and then less than 40-ratio of ‘5, 20, 19, 7, 1, 8, 13, 10, 14’. This measure favors more the sequenced correlation with area than perimeter, while also tying with the latter in narrowest variance. It performs another interactive role with the perimeter and circumcircle in one coherent structure of geometrical clustering. The specific triangle (1, 2, 12) of ‘Tahrir’ continues the change of dynamic clustering with roundabout Cairo, but the other two triangles continues the stable clustering throughout the geometrical measures. The same triangle measures closest to the Delaunay graph in overall average values.

TABLE 2: Delaunay Spatial Measures

Gen.	RA	Control	Degree	Indirect E
1	0.1538	0.9	4	0
2	0.1208	1.0095	5	1
3	0.1318	0.6761	4	1
4	0.0989	1.2595	6	2
5	0.1428	0.7095	4	1
6	0.1428	1.2	5	3
7	0.1758	0.6	3	0
8	0.1318	1.1261	5	2
9	0.0879	1.5	7	3
10	0.1318	1.1261	5	1
11	0.1868	0.65	3	1
12	0.1208	1.6166	6	3
13	0.2197	0.4166	2	1
14	0.1428	1.0333	4	3
15	0.1428	1.2095	5	1
Ave.	0.1421	1.0021	4.533	1.533

Further space syntax of the Delaunay graph measures the integral and control structure, with graph accounts “Table 2”. The most integral vertices of generators ‘9 & 4’ correspond to the first ‘Hippodrome’ of large-scale public facility and the royal ‘Abdean Palace’ respectively. The vertices at the pole of ‘Ezbekia Garden’ have less integration than control, which enforces the filtration process with old Cairo adjacency. However, ‘Tahrir’ pole has more integrity with less control, except generator-12 of highest control that extends royal premises along the Nile. Meanwhile, generator-1 of ‘Tahrir’ itself with generator-8 of the ‘Opera’ at ‘Ezbekia Garden’ exchange their clusters of syntactic measures of close average to the Delaunay’s total average. Meanwhile, the least integral and control syntax of generator-11concentrates unpleasant workshops. The residential function generates suburbs of autonomous high control and less Delaunay integrity of infinite ‘garden city’ extends. The algorithmic graph observes the cyclic Delaunay structure, which satisfies the ‘Hamiltonian’ property to visit every vertex once in line. However, the graph is not ‘Eulerian’ to traverse all edges without re-traversing any of them due to the degree measure of more than two odd vertices. In spatial terms, the Delaunay graph of roundabout Cairo is planned as one connected network to cross them all in one round of selected edges or boulevards, but not necessarily to pass by all edges once in a round-trip. Meanwhile, the graph is free of cut vertex or edge where it is possible to skip any of them without having to alter the network. The average degree of five edges per vertex enforces the radial concept of roundabouts. Meanwhile, the indirect access of vertices has the least encounters of average 1.5-edges, which maximizes the probability of boulevard connectivity for the roundabout structure in reality. The graph property of 'five-choosable' coloring for the triangulated Delaunay graph with cycle bound optimizes the spatial variety of choice. Accordingly, the alternative choice of vertices with disjoint paths minimizes the graph separation.

The map of Haussmann’s roundabout planning with the space syntax and algorithms of the Delaunay graph observes several correlations. The degree of generator-9 measures optimal 7-edges of only 4-boulevards on ground. Accordingly, the top integral space syntax of the 'Hippodrome' structures a half boulevard probability with high control. The logic behind this spatial structure observes the minimized through-pass for the low-density residential area where the 'Hippodrome' is centrally located, while connecting its roundabout for public access. On the contrary, generator-13 minimizes in 2-edges and 5-boulevard connectivity, where the elevated plateau of the 'Citadel' logically compensates the drop of integration and control syntax by boulevard realization down on ground. Similarly, generators '7 & 11' of common 3-edges but more boulevards, together with the 'Citadel', set a cluster of the least space syntax measures. Meanwhile, generator-8 of recessed roundabout from boulevards, but with 5-edges of graph connectivity, has an average space syntax of integral and control measures. The comparable recession of generator-15 from boulevards with graph of 5-edges has less integrative and more controlling measures. Both generators form a drop-off circulation rather than boulevard connectivity, which suits their public function of the ‘Opera House’ and the ‘Cairo Railway Station’ respectively. Overview of major public buildings miscorrelate between boulevards and graph edges in logic with space syntax measures to keep integrated at the global spatial system with exclusive roundabout circulation on ground. Nevertheless, the most balanced structure of generators ‘1, 2 & 10’ of five-edges in graph corresponding to five-boulevards in map with space syntax average measurements further enforce the analogical roundabout structure of ‘Tahrir’ pole.

3. Conclusion

The geometrics of Delaunay diagram with space syntax and graph algorithms determine the spatial structure of roundabout Cairo in 1867 for conservation as follows:

1. The circumcircle property of Delaunay triangulation forms a cluster of dense corridor with superimposed boulevard projections from the roundabouts of ‘Ezbekia Garden’ towards the historical landmarks of ‘El Azhar Mosque’ and ‘Cairo Citadel’, in addition to ‘Cairo Station’ of regional railway connections. The opposite end of ‘Tahrir Roundabout’ projects over the widest portion of the Nile towards the Giza Pyramids. The largest circumcircles, nevertheless, spread over the crossing axis with the spanning of Cairo beyond the new European Quarter limits, while the intermediate circumcircling links the two extreme sizing in overlap.
2. The geometrical dimensions form a dynamic clustering of Delaunay triangulation, which structures the degree angles into zoned clusters of maximum and minimum ranges in correlation to circumcircles, but large intermix of moderate measures. The length of triangular sides re-clusters into random shortest lengths, with more correlation to the circumcircles at medium and longest measures. The perpendicular distance from vertex to edge again re-clusters into stable correlation with the point distribution, though of some variance among the medium measures. The triangular areas correlate with the point spacing, however with change in clustering sequence. The perimeters correlate in more order with circumcircles than areas, while the ratio of area to perimeter favors the order of areas more than circumcircles. In this dynamic structure, the triangular measures of ‘Tahrir’ perform the most stable correlations with the Delaunay clustering and overall average.
3. The Delaunay graph measures the space syntax of integration and control, which observes the dominance of public and royal generators in spatial measurements. The less integrity of ‘Ezbekia Garden’ filters from old Cairo interface, with more integration at ‘Tahrir’ opposite pole. The least integral and control measures hide the bulk of workshops from the public, while the freed ‘garden city’ suburbs secure high control generators. The average space syntax matches with ‘Tahrir’ in common with the ‘Opera House’ of ‘Ezbekia Garden’ to enforce their polar structure. Further graph algorithms prove ‘Hamiltonian’ but not ‘Eulerian’ properties with maximal coloring opportunities and minimal indirect access of the Delaunay vertices, which enforces the radial concept of roundabouts. This is proved by the real map of Haussmannized Cairo in the corollary between the degree of vertices and the number of radial boulevards, with the exact match in genetic ‘Tahrir’.

4. Acknowledgements

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