$$
\begin{aligned}
&\sum_{n=0}^{N}\left[(-1)^n W_{4n}-W_{2n}\right]=0,\sum_{n=0}^{N}\left[(-1)^n W_{4n}-W_{3n}\right]=0,\sum_{n=0}^{N}\left[(-1)^n \Psi_{4n}-\Psi_{2n}\right]=0,\sum_{n=0}^{N}\left[(-1)^n \Psi_{4n}-\Psi_{3n}\right]=0,\\
&\sum_{n=0}^{N}\left[(-1)^n U_{4n}-U_{2n}\right]-e_2\sum_{n=1}^{N}(-1)^n \Psi_{4n}=0,\sum_{n=0}^{N}\left[(-1)^n U_{4n}-U_{3n}\right]+e_3\sum_{n=1}^{N}(-1)^n \Psi_{4n}=0,\\
&\sum_{n=0}^{N}\left[A_{551}(-1)^n \Psi_{4n}-A_{552}\Psi_{2n}-A_{553}\Psi_{3n}\right]+\frac{2A_{551}}{L_4}\sum_{n=1}^{N}W_{4n}\sum_{k_1=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}(2n-4k_1-1)(-1)^{n-2k_1-1}-\sum_{n=1}^{N}\left[\frac{2A_{552}}{L_2}W_{2n}+\frac{2A_{553}}{L_2}W_{3n}\right]\sum_{k_1=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}(2n-4k_1-1)=0,\\
&\sum_{n=1}^{N}\left(\frac{A_{111}}{L_4}U_{4n}+\frac{B_{111}}{L_4}\Psi_{4n}\right)\sum_{k_1=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}(2n-4k_1-1)(-1)^{n-2k_1-1}-\sum_{n=1}^{N}\left[\frac{A_{112}}{L_2}U_{2n}+\frac{B_{112}}{L_2}\Psi_{2n}+\frac{A_{113}}{L_2}U_{3n}+\frac{B_{113}}{L_2}\Psi_{3n}\right]\sum_{k_1=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}(2n-4k_1-1)=0\\
&\sum_{n=1}^{N}\left(\frac{B_{111}}{L_4}U_{4n}+\frac{D_{111}}{L_4}\Psi_{4n}\right)\sum_{k_1=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}(2n-4k_1-1)(-1)^{n-2k_1-1}+\\
&\sum_{n=1}^{N}\left[\left(\frac{A_{112}e_2}{L_2}-\frac{B_{112}}{L_2}\right)U_{2n}+\left(\frac{B_{112}e_2}{L_2}-\frac{D_{112}}{L_2}\right)\Psi_{2n}-\left(\frac{A_{113}e_3}{L_2}+\frac{B_{113}}{L_2}\right)U_{3n}-\left(\frac{B_{113}e_3}{L_2}+\frac{D_{113}}{L_2}\right)\Psi_{3n}\right]\sum_{k_1=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}(2n-4k_1-1)=0
\end{aligned}
\tag{A-6}
$$

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