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Teaching and Learning by using GeoGebra

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Abstract: *When considering a human model, the model must be viewed as a process model, as a knowledge model, and as a control model. We can consider that such an approach has been applied to the understanding of theorem proving. Humans appear to use two types of knowledge to understand, investigate and act. The human model for understanding in mathematical thinking is based on a three-level model of human action. In cognitive science, human beings use strategies to solve problems. Strategies are also used when human beings solve mathematical problems. We used a three-level human behavior model to analyze the targets that appear during problem solving and the strategies used to solve the problems.*
GeoGebra with a theorem prover is being developed. We must analyze the GeoGebra as prover of cognitive science.

Keywords: *Pedagogical model, Cognitive science, proving, GeoGebra.*

1. Introduction

Human modeling can elucidate how humans make decisions and how learning ability can be improved. Techniques that have been used widely in this field can be classified into those that integrate a control theory model [20], a probability theory model [1], a will determinism model [6], an information processing model [2], or an artificial intelligence model [11], and those integrating all of these models [18]. The rapid development in research on computer technology, cognitive psychology, and artificial intelligence has led to a shift from behavioristic models [16], which ignore the psychological processes of behavior, to information processing models [9] and artificial intelligence models.

However, these models are still limited with respect to human cognition and while promising new research is being conducted, such research [3] has not been collated into a theoretical system with specific methods. When considering a human model, the model must be viewed as a process model, a knowledge model, and a control model. At present, the models used to describe human information processing include a 7-step user action model, a 3-level human action model, a human cognition model, and a knowledge model. Unfortunately, there is currently no optimal model of understanding in cognitive science. When constructing a new model of human cognition, the effects of various factors should be accounted for. Attempting to construct a model based on the consideration of as many factors as possible would complicate the model with too many hypotheses and parameters, making it difficult to test.

Therefore, the human model proposed in the present work is based on the three-level model of Rasmussen [14] arose from research on human actions in large-scale systems such as an atomic energy plant or an aircraft. We can consider that such a model has been applied to understanding in cognitive science. Thus, this model was used to position problem-solving strategy and clarify cognitive changes. Almost all human cognitive activities arise from an interactive process between information from the outside world and the knowledge possessed by the individual. Information “from the outside world” is affected by the cultural background and social customs. ”Knowledge possessed by the individual” is influenced by the experiences of the individual. Despite this complicated and continuous interactive reaction between external and internal factors, there are environments and stimuli common to all humans; common human reactions exist. Internal knowledge includes not only linguistic and analytical intellectual knowledge, but also non-linguistic and comprehensive intellectual knowledge. Humans seem to use both types of knowledge to understand, learn, and act. Determining whether such human activities can be accurately evaluated and included in a theoretical framework is the key to developing a user interface that can elicit the full intellectual potential of humans.

If we want to conduct effective cognitive science that identifies the source of understanding and creativity, rather than simply utilize unconscious human actions to reach into the depths of human psychological activities, then we must establish a framework that explains the communication between the external and internal sources of knowledge. Our proposed human model of understanding in cognitive science is based on a three-level model of human action. Human researching begins with physical processes, followed by visual processes, and finally intuitive (symbolic) processes. In rule-based actions, classifying behavior occurs in response to problems. This makes efficient search possible, by indicating what should be done next based on the present situation. To examine how the classification is expressed and constructed, subjects are given several problems to solve for the present study. If the calculation procedure is incorrect when the behavior classification is being constructed, the results will not be predictable. What required identification was how easy it was to reconstruct the classification and what conditions facilitate the identification of the actions that were the cause of the error. In order to assess the degree of freedom achieved, a measurement method was required for identifying the necessary conditions for moving from a rule-based activity to a skill-based activity. Knowledge-based behavior results from cognition and interpretation of external conditions and the construction of a psychological model that uses skill- and rule-based behavior as a solution process.

This requires knowledge of how humans solve problems. Information must be organized and recorded for cognition and understanding. Receiving a message means that the message is reconstructed. In order to do this, one must have the resources with which to conduct this reconstruction. An agent should be able to function appropriately with human common sense and the ability to learn. The agent should always consider safety and accuracy and also possess the ability to explain what is dangerous. The issues in designing such an agent and its actual use must be identified. Mistakes in knowledge-based behavior due to illusions or uncertain memory can lead to the inability to choose or the lack of knowledge of an operation procedure. To integrate these skill-, rule-, and knowledge-based behaviors, the three-level human action model was painstakingly reconstructed in order to establish a useful human model, which is used for the learner in cognitive science.

Gaining knowledge and solving problems is two side of the same coin. Reasoning is a reduction of the complex to the simple. Mathematics is the science of "explanation" (reducing everything to "plain things"). By using theorem prover, we can reduce the proofs of complicated theorems to simple steps. The development of theorem prover is an exciting study of mathematics and computer science. Creating the mathematical technology (creating the algorithm) proceed by thinking and applying the results. There are many computer algebra systems for proving mathematical theorems in automatic mode. In this paper, we will show the structure of GeoGebra.

2. Human Model

According to the three-level human behavior model of Rasmussen, automatic human actions can be classified into the three levels of skill-, rule- and knowledge-based actions (Fig.1, [15]).

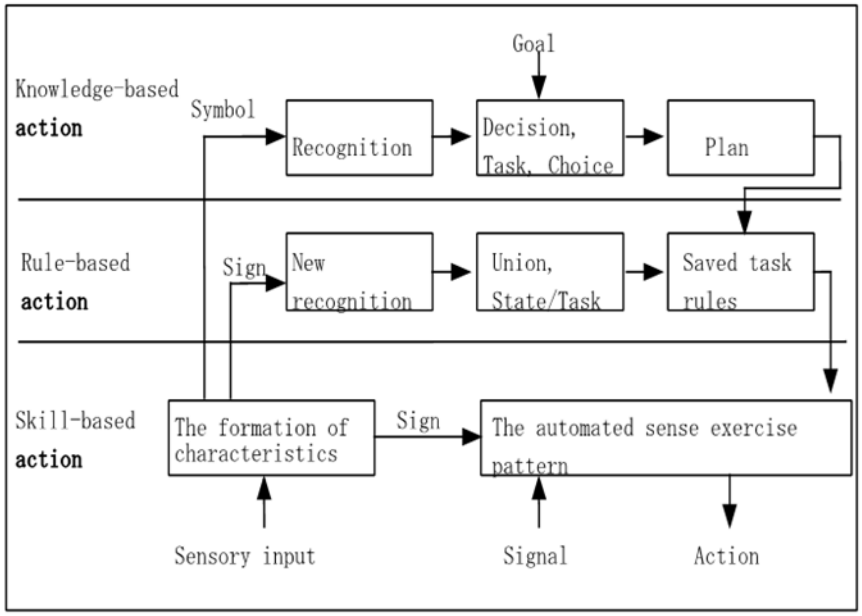

Fig. 1. Three-Level Model of Human Action

Skill-based action: These actions occur at the stage when some intention is formed. However, the actions are automated and are executed without the control of an action pattern.

Rule-based action: Actions that utilize a previously acquired rule in order to realize a specified purpose. The learner constructs and executes a series of actions.

Knowledge-based action: The learner recognizes the situation, manipulates a psychological model, and considers possible strategies.

A skill-based action is a response that occurs in less than 1 second [12]. A chain of skill-based actions is a rule-based action. Thinking about how to solve a problem is a knowledge-based action. Skill-based actions are performed smoothly without intentional control. Rule-based actions require a great deal of repetitive practice in order to be transferred to the skill-based level. First, the external conditions must be recognized, then the rules for composing the act are combined with the conditions required to carry out the behavior. Knowledge-based actions require the recognition of external conditions, the interpretation of these conditions, the construction of a psychological model for considering solutions, planning, and finally, the use of the other two behavior levels to carry out the action. This is a process model in which mastery of behavior requiring thought is internalized to the point where it can be carried out unconsciously. Mistakes can be explained as omitted steps, or for example, as pushing the wrong nearby button in smoothly carried out skill-based actions. In the case of knowledge-based actions, illusion can lead to error. In the present study, this process was analyzed using Rasmussen's three-level human behavior model in order to identify what functions are essential to facilitating smooth action and learning. Behavior used to learn about problems and how to solve them is classified in detail according to the three-level model. Humans act by classifying issues and their relationships by consciously combining them. Humans control themselves by constantly observing, thinking about, evaluating, and integrating their behavior in order to achieve accuracy, continuity, consistency, and normality [8]. Classified factors can be separated into the same three levels as the general actions.

3. Strategy

In cognitive science, humans use strategies to solve problems. Strategies are used as knowledge to plan solutions and decide procedures. When these procedures, in general or for the most part, obtain the correct answer, the procedure is called a heuristic; however, such heuristics do not always result in a correct solution. The study of factors related to the early stages of strategy is based on observing the results of an individual’s problem solving method. Following the initial stage, the individual passes through a series of stages and then reaches a solution. However, consensus has not been reached regarding the meaning of these stages. For example, Wallas identified the following four stages involved in problem solving in the book “The Art of Thought” [19]:

1. Preparations (collecting information)
2. Warming up (Separating from the problem)
3. Flash (Sudden insight)
4. Verification (Reaching a solution)

Duncker's observation that “A person resolves the problem into a goal for the lower rank, and then he tries to solve the goal for the lower rank” [4] could not be verified. At this time, not much could be inferred about human accomplishments, and an individual’s thought processes could not be described clearly. Subsequently, cognitive psychology came to the forefront, and Polya clarified a method for teaching problem-solving strategies in [13]. Ernst and Newell, and later Newell and Simon, extracted strategies using the following method [10]. A man involved in solving a problem was directed to communicate the contents of his decision-making process and his thoughts through speech. The strategy used by the man was extracted from the records of his speech. Newell designed the General Problem Solver (GPS) computer program based on the problem-solving methods that were clearly and accurately described. This GPS successfully solved the following 11 problems, which are all different kinds of problems.

1. Missionary and cannibals (Three missionaries and three cannibals must cross a river using a boat that only two people can ride at a time)
2. The integral calculus (The application of an integral formula)
3. The tower of Hanoi (Different size disks must be moved and placed on three sticks)
4. The proof of the theorem (The theorem to solve a function calculation)
5. Father and sons (A father and 2 sons cross a river on a boat with a 200-pound-capacity)
6. Subject of the monkey (There is a banana at a height that a monkey can not reach)
7. Three coins (Two coins can be overturned at a time, and by the third attempt all coins must be showing the same side)
8. Sentence analysis (Distinction of parts of speech)
9. Water pot problem (Using five gallon and eight gallon water tanks two gallons of water must be measured)
10. Character line complete problem (The rule must be detected from a row of characters)
11. The bridge of Kernihisberg (When returning to the city, one passes seven cities and seven bridges, but passes each bridge only once)

For a computer to solve a problem, the expression of the problem solution strategy must be accurately described as follows:

1. The problem must be expressed as a problem space.
2. The problem space consists of the possible operations available to change the premise conditions of the problem, the final conditions of the problem, and the middle conditions.
3. The problem space has a passage that can be used (dead end).
4. Solutions to the problem are to be sought from the viewpoint of problem-solving goals. They are not carried out from a trial and error perspective.

Strategies are used even when human beings solve mathematical problems. Recognition knowledge and experience are used as “doing it like this is effective in this case”. The ability to rapidly reference knowledge is required for strategies based on experience. Furthermore, the recognition of thoughts and feelings controls. The famous book by the mathematician Polya, “How to solve it [13], showed the processes of mathematical problem solving; however, one can not learn how to use heuristics in problem solving just by reading a book. In researching problem solving, there are two contrasting concepts. The first emphasizes insight, flash, and senses, while the second emphasizes experiential knowledge. The former concept employs a strong tendency to perceive that strategies of thought are learned through the experience of problem solving. In other words, it is assumed that an intuitive feelings and specific technical abilities can be acquired. In the latter concept, it is assumed that problem solving ability arises from the accumulation of rules inherent to the domain provided by an individual problem. Such differences depend on the problem's nature, domain, and level, and the type of person involved in the learning process. In addition, it is difficult to establish clear boundary lines between these two concepts. In problem solving, experiential knowledge plays a large role. Heuristics are general ideas or algorithms (a procedure providing the correct solution), and are widely used. Heuristics are equal to "the logic of a thought". Examples of extremely general strategies are "try to draw a figure if you come across a difficult problem", and "search for similar problems that you have experience with". There are also concrete strategies we are familiar with, such as "A problem requiring the comparison of quantities requires two differences, and a transform formula" and "try to make clauses that differ next to each other for number sum sequence problems" [5]. Therefore, the kinds of strategies that students actually use were examined. The present research explores the differences in the learning of students who adopt the strategy of physical principles without knowledge of the usual learning methods for mathematics and students who adopt the natural strategy of mathematics, which proceeds with logical, progressive thought. These differences are assembled in the student's brain, which determines how these strategies are to be used. Finally, a learner independently searches for heuristics. It is desirable to understand this process, in order to recognize the meaning of cultivating natural mathematical comprehension and thinking power. However, currently, there are many cases in which the learner does not consciously study problems from the perspective that "there are various learning methods and heuristics".

4. Theorem Prover

The theorem prover are being used frequently in mathematics teaching. Although good teaching examples and experiences exist, the efficient and successful use is not self-evident, yet. A subtle relationship exists between paper-and-pencil techniques, theorem prover, and conceptual understanding.

The nature of theorem prover is different from that of paper-and-pencil techniques. Mathematics are presented as a primarily mental activity that involves the construction of mathematical objects and relations. Using the theorem prover requires insight into procedures as well as into the concepts involved.

In cognitive science, use of a computer with the theorem prover can correct the weakness in mathematical thinking. We can clearly understand mathematical concepts and can minimize the burden of operation opportunities. Computer software using a theorem prover have bring about changes in mathematical thinking. Therefore, learners can concentrate on mathematical problems.

GeoGebra: GeoGebra is an interactive geometry, algebra, statistics and calculus application, intended for learning and teaching mathematics and science from primary school to university level. GeoGebra is an interactive mathematics software program for learning and teaching mathematics and science from primary school up to university level. Constructions can be made with points, vectors, segments, lines, polygons, conic sections, inequalities, implicit polynomials and functions. All of them can be changed dynamically afterwards. Elements can be entered and modified directly via mouse and touch, or through the Input Bar. GeoGebra has the ability to use variables for numbers, vectors and points, find derivatives and integrals of functions and has a full complement of commands. Teachers and students can use GeoGebra to make conjectures and to understand how to prove geometric theorems. GeoGebra [21] is a good platform for experimentation, which supports the development of mathematical concepts and the abilities to explain geometric properties. By using GeoGebra, we can aim at consolidating students' understandings on the geometrical concepts through experiencing the process of exploring, conjecturing, verifying, justifying and proving is described.

5. Knowledge Base in Cognitive Science

When researchers use the theorem prover for the acquisition of knowledge or skills, we must consider a “tool” to be a “symbol device”. A symbol device exists between the researchers and the research subject. Operation activity occurs between a symbol device and the researching subject. In cognitive science, two difficulties exist, one in the interaction between the researcher and the symbol device, and one in the interaction between the symbol device and the research subject. Therefore, we must overcome these difficulties in order to effectively utilize the theorem prover in cognitive science. Moreover, we must assess the benefits of considering the integration of the theorem prover from the perspective of the relationship between mathematical knowledge and mathematical concepts. When theorem provers are used in mathematical studies, researchers achieve a result through their efforts. Then, the researchers must investigate whether conceptual problems exist or whether they simply do not appreciate how the theorem prover works. By using a theorem prover effectively, researchers become aware of numerous mathematical ideas. This is made possible by incorporating the results of research in cognitive science. In carrying out a seven-phase model of human action, “the formation of a series of intentions or actions” must be performed smoothly [17]. The effective use of a theorem prover in cognitive science is influenced by the contents of mathematical thought, and research and understanding of mathematics can further influence general idea formation. The theorem prover influences the “perception - interpretation - evaluation” phases of evaluation. The foundations of this model were studied by Rasmussen as the three-level control model of individuals’ actions [17]. We can use the theorem prover as a material object that is available for the assessment of human activity. The use of the theorem prover can establish automatic and routine procedures. Controlling this automation is essential, especially in research on though processes. There are three methods for creating a theorem proof (by hand, by mind, and with a computer). A researcher's point of view of cognitive science considers the relationship between the brain and mind as the relationship between hardware and software in a computer. According to this point of view, the science of the mind is a special science, the science of thought.

We must distinguish between a tool and an instrument. Recent studies have given us theoretical tools for improved understanding of human/machine interactions. The psychological component is defined through the notion of a scheme. Operations rely on implicit knowledge, specifically, concepts that are implicitly considered. This knowledge guides the action. A scheme is itself the product of an assimilation activity, in which the computer and the available theorem prover play major roles. An instrument is a mixed entity, with a material component (a tool, or a part of a tool activated in order to realize a specific type of task) and a psychological component (the schemes organizing the activity of the subject). The theorem prover can act on several levels as an instrument in cognitive science:

- The first level (the level of the default system);
- The second level of an instrument or a set of instruments;
- The third level (a meta one) of the relationship of a subject with an instrument or a set of instruments.

These three levels correspond to tools in different levels of cognitive science:

- Primary tools, the concept of the tool for initial research;
- Second-level tools, representations, and action modes utilizing first level tools;
- Third-level tools, for trained researchers.

6. Conclusion

In the three-level model of human behavior, operations and strategies can be identified and considered in relation to human thought processes in order to facilitate error-free problem solving. In consideration of surface features and conditions, similar problems can be recognized and suitable problem-solving methods can be identified. In addition, it was found that contents of the subconscious can be raised to the knowledge-based action level in order to support the expression process and the achievement of efficient functioning.

The technology of theorem prover automated reasoning. The ultimate goal of mathematics is technology. To do mathematics is gaining knowledge and solving problems by reasoning. Theorem prover is a powerful tool for researching mathematics. Researchers should appreciate the possibility of sharing cognitive level with such technology (Fig. 2).

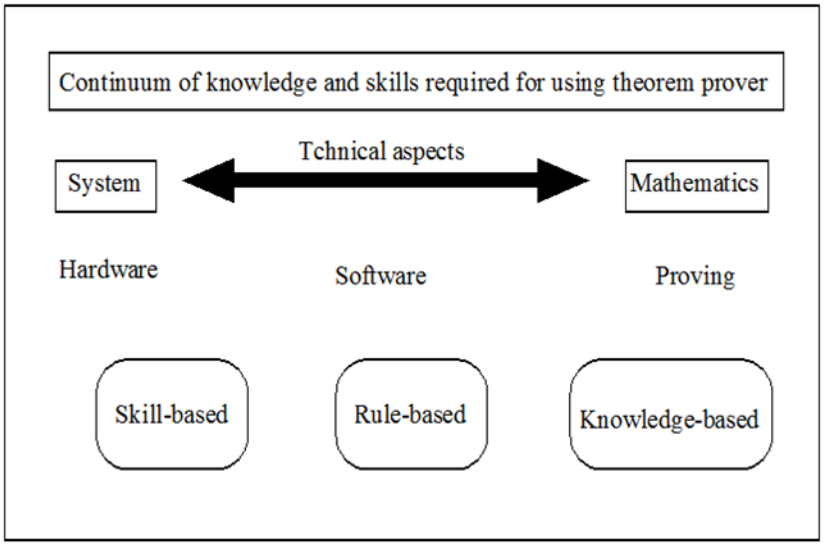

Fig. 2. The continuum of knowledge and skills required for using theorem prover

Researchers can use theorem prover as a 'partner'. However, researchers must study not to reach for the computer in an unthinking manner. Researchers need to be aware that algebraic expressions using theorem prover syntax may be time consuming. If an expression is to be used repeatedly in the proof of a theorem then this initial time spent on theorem prover entry may be worthwhile but researchers will require guidance in making these choices. In contrast, strategic use requires the researcher to think about the mathematics and the tool. The value of theorem prover will be realised once researching with the tool simulates rather than impedes mathematical thinking. To reach this stage, researchers must encourage to develop the habits of discerning and strategic use of theorem prover along with the elements of algebraic insight required. In this way, theorem prover gives many opportunities for rich mathematical researching. Technology in mathematics is the fulfilment of the aspiration of mathematics in the computer age. By recent advances in computational logic, computational algebra and software technology, the automation of reasoning promises to become practical feasible and useful for mathematics research.

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